Thursday, September 03, 2009
3:34 PM

Administrativa

1. My office hours: M 3-4 pm
2. Refrain from sending me code
   if you need help with C++, i.e.
   compiling, get a tutor

Thursday, September 03, 2009
4:06 PM

Last time: 2 methods for solving recurrence relations

Method 1: telescoping sum, e.g. for mergesort

$T(1) = 1$

$T(n) = 2T(\frac{n}{2}) + n$ N

divide by n

$$\frac{T(n)}{n} = \frac{2T(n/2)}{n} + 1 = \frac{2}{n} \cdot T(\frac{n}{2}) + 1 = \frac{T(\frac{n}{2})}{\frac{n}{2}} + 1$$

telescope out
trying to reduce
n down to 1

$$\frac{T(\frac{n}{2})}{\frac{n}{2}} = \frac{T(\frac{n/2}{2})}{\frac{n/2}{2}} + 1 = \frac{T(\frac{n}{4})}{\frac{n}{4}} + 1$$

$$\frac{T(\frac{n}{4})}{\frac{n}{4}} = \frac{T(\frac{n}{8})}{\frac{n}{8}} + 1$$

$$\vdots \qquad \vdots$$

$n=2$ $$\underbrace{\frac{T(2)}{2}}_{\text{add up}} = \underbrace{\frac{T(1)}{1}}_{\text{add up}} + 1$$

add it all up (telescoping sum)

$$\frac{T(n)}{n} + \cancel{\frac{T(n/2)}{n/2}} + \cancel{\frac{T(n/4)}{n/4}} + \cdots + \cancel{\frac{T(2)}{2}} =$$

$$\cancel{\frac{T(n/2)}{n/2}} + 1 + \cancel{\frac{T(n/4)}{n/4}} + 1 + \cancel{\frac{T(n/8)}{n/8}} + 1 + \cdots + \frac{T(1)}{1} + 1$$

$$\boxed{\frac{T(n)}{n}} = \sum_{\lg n} 1 + \frac{T(1)}{1} = \lg n + \frac{T(1)}{1}$$

why $\lg n$?

$$T(n) = n\lg n + n$$

$$\in O(n \lg n)$$

let's say $n = 8$

$\frac{T(n)}{n} = \frac{T(8)}{8}$ $\quad 2^{3}$

$\frac{T(n/2)}{2} = \frac{T(4)}{4}$ $\quad 2^{2}$

$\frac{T(n/4)}{4} = \frac{T(2)}{2}$ $\quad 2^{1}$

$\frac{T(n/8)}{8} = \frac{T(1)}{1}$ $\quad 2^{0}$

Method 2: recursive back substitution

Thursday, September 03, 2009
4:23 PM

$T(1) = 0$

$T(n) = 2T\left(\frac{n}{2}\right) + n - 1$

level 1: $T(n) = n - 1 + 2T\left(\frac{n}{2}\right)$

level 2: $= n - 1 + 2\left(\frac{n}{2} - 1 + 2T\left(\frac{n}{4}\right)\right)$

$= n - 1 + n - 2 + 4T\left(\frac{n}{4}\right)$

level 3: $= n - 1 + n - 2 + 4\left(\frac{n}{4} - 1 + 2T\left(\frac{n}{8}\right)\right)$

$= n - 1 + n - 2 + n - 4 + 8T\left(\frac{n}{8}\right)$

level 4: $= n - 1 + n - 2 + n - 4 + 8\left(\frac{n}{8} - 1 + 2T\left(\frac{n}{16}\right)\right)$

$= n - 1 + n - 2 + n - 4 + n - 8 + 16T\left(\frac{n}{16}\right)$

$\vdots$

level k: $= \sum_{i=0}^{k-1} n - \sum_{i=0}^{k-1} 2^i + 2^k T\left(\frac{n}{2^k}\right)$

look up sheet with handy formulae

$\sum_{i=0}^{n} k = k(n+1)$

$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

want this to be $T(1)$

$n(k-1+1) - \left(2^{k-1+1} - 1\right) + 2^k T\left(\frac{n}{2^k}\right)$

$= kn - 2^k + 1 + 2^k T\left(\frac{n}{2^k}\right)$

let $2^k = n \Rightarrow \underline{k = \lg n}$

$$= n\lg n - n + 1 + nT\left(\frac{n}{n}\right)$$

$$= n\lg n - n + 1 + \cancel{nT(1)}$$

$$= n\lg n - n + 1 = O(n\lg n - n) \in O(n\lg n)$$

Thursday, September 03, 2009
4:35 PM

Handy formulae

$$1+2+3 \cdots + n$$
$$n+(n-1)+(n-2) \cdots +2+1$$
$$(n+1)+(n+1)+ \cdots + n(n+1)$$

$$\sum_{i=0}^{n} k = k(n+1) \qquad \sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1}-1 \qquad \sum_{i=0}^{n} i2^i = (n-1)2^{n+1}+2$$

$$\sum_{i=0}^{\lg n-1} 2^i = n-1 \qquad \sum_{i=0}^{\lg n-1} n\lg\frac{n}{2^i} = \frac{n\lg^2 n}{2}+\frac{n\lg n}{2}$$

For homework:

review Quicksort (worst case & best case)

$T(0) = T(1) = 1$

$T(n) = T(n-1) + cn$ $\qquad$ $T(n) = cn + 2T\left(\frac{n}{2}\right)$

$k=0$ $\qquad = cn + T(n-1)$