Today:

1. inductive proof "rule of thumb",

   a) assume assertion holds for n

   b) show it holds for $n+1$

   (see online notes — they were updated from last time)

2. Theory: recurrence rel.

3. Practice: timer_t, asg0 (omp)

- generally, types of algs we'll see involve splitting a list in half (divide & conquer)

e.g.,

```
insert(list)
{   if(list.size()==1)     - O(1)
        return;
    //split list in 2      - O(n)
    insert(list/2)         - O(n/2)
    insert(list/2)         - O(n/2)
}
```

$O(1)$

$O(n)$

$O(\frac{n}{2})$

$O(\frac{n}{2})$

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

$$T(1) = 1$$

Solve: 2 methods:

telescoping sum

recursive back sub.

e.g. Telescoping sum (mergesort)

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$

divide by n

$$\frac{T(n)}{n} = \frac{2T\left(\frac{n}{2}\right)}{n} + 1$$

$$= \frac{T\left(\frac{n}{2}\right)}{\frac{n}{2}} + 1$$

$$\frac{T(n)}{n} = \frac{T(n/2)}{n/2} + 1$$

valid for any n a power of 2

$$\frac{T(n/2)}{n/2} = \frac{T(n/4)}{n/4} + 1$$

$$\frac{T(n/4)}{n/4} = \frac{T(n/8)}{n/8} + 1$$

$$\vdots$$

$$\frac{T(2)}{2} = \frac{T(1)}{1} + 1$$

add it all up (telescoping <u>sum</u>)

sum:

LHS: $\frac{T(n)}{n} + \cancel{\frac{T(n/2)}{n/2}} + \dots, \cancel{\frac{T(2)}{2}}$

$=$

RHS: $\cancel{\frac{T(n/2)}{n/2}} + 1 + \cancel{\frac{T(n/4)}{n/4}} + 1 + \dots +$

$\frac{T(1)}{1} + 1$

$$\frac{T(n)}{n} = \sum_{\log_2 n} 1 + \frac{T(1)}{1}$$

$$\frac{T(n)}{n} = \sum_{\lg n} 1 + \frac{T(1)}{1}$$

mult. by n to get $T(n)$

$$T(n) = n\Big(\underbrace{\sum_{\lg n} 1}\Big) + n$$

$$n(\lg n) + n$$

$$\in O(n \lg n)$$

Method 2 : recursive back sub.

$T(1) = 0$

$T(n) = 2T\left(\frac{n}{2}\right) + n - 1$

level 1: $T(n) = 2T\left(\frac{n}{2}\right) + n - 1$

$= n - 1 + 2T\left(\frac{n}{2}\right)$

level 2:

$= n - 1 + 2\left(\frac{n}{2} - 1 + 2T\left(\frac{n}{4}\right)\right)$

$= n - 1 + n - 2 + 4T\left(\frac{n}{4}\right)$

level 3: $n-1+2T(\frac{n}{2})$ from for

$= n-1+n-2+4\left(\frac{n}{4}-1+2T\left(\frac{n}{8}\right)\right)$

$= n-1+n-2+n-4+8T\left(\frac{n}{8}\right)$

---

level 4:

$= n-1+n-2+n-4+n-8+16T\left(\frac{n}{16}\right)$

$\vdots$

level K:

$$= \sum_{i=0}^{K-1} n - \sum_{i=0}^{K-1} 2^i + 2^K T\left(\frac{n}{2^K}\right)$$

$T(n) =$

$$\sum_{i=0}^{k-1} n + \sum_{i=0}^{k-1} 2^i + 2^k T\left(\frac{n}{2^k}\right)$$

want $T(1)$

to get $T\left(\frac{n}{2^k}\right)$ to be $T(1)$

let $2^k = n$ so that $k = \log_2 n$

$T(n) =$

$$\sum_{i=0}^{k-1} n + \sum_{i=0}^{k-1} 2^i + \underbrace{nT\left(\frac{n}{n}\right)}_{nT(1)}$$

$$\sum_{i=0}^{k-1} n$$ look this up

$$\sum_{i=0}^{n} k = k(n+1)$$

$$n(k-1+1) = nk$$

$$\sum_{i=0}^{k-1} 2^i$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

$$2^{(k-1+1)} - 1 = 2^k - 1$$

but $2^k = n$

$T(n) =$

$(\lg n)n - n + 1 + \cancelto{\emptyset}{nT(1)}$

$= n\lg n - n + 1$

$\in O(n \lg n)$

Handy formulae:

$$\sum_{i=0}^{n} k = k(n+1)$$

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$