Quicksort

Alg (pseudocode, sort of):

Indices

```
quicksort(array, left, right)
  if (right > left) {
    //pick a pivot
    pivot = partition(
        array,
        left, right,
        pivot)
```

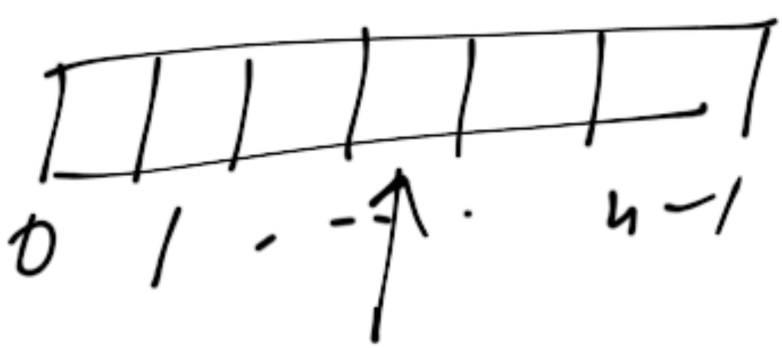
also an index, value can change

quicksort(arr, left, pivot - 1)

quicksort(arr, pivot + 1, right)

} //end of if

- picking a pivot:




integer div

pivot = left + (right - left)/2

Quicksort analysis:

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + \underbrace{T(\text{partition})}_{n}$$

partition(arr, left, right, pivot)

1. store data at pivot location
   the_pivot = arr[pivot]

2. move pivot element to end of array
   swap(arr[pivot], arr[right])

3. go thru arr from left to right moving values > the_pivot to the left of pivot index, advancing left index

```
for(i=left; i<right; i++)
  if (arr[i] <= the_pivot){
     swap(arr[left],arr[i]);
     left++;
  }
}
```

4. move pivot element to its final location

```
swap(arr[left], arr[right]);
```

5. return pivot's new location

return (left)

(note: 'left' must be pass-by-value)

8

3 7 8 $\boxed{5}$ 2 1 9 5 4

^

left pivot

i

3 7 8 4 2 1 9 5 $\boxed{5}$

left

i . . . . . i

3 4 8 7 2 1 9 5 $\boxed{5}$

left i

3 4 2 7 8 1 9 5 $\boxed{5}$

left i

3 4 2 1 8 7 9 6 $\boxed{5}$

left i

3 4 2 1 5 7 9 8 $\boxed{5}$

left

// end of for loop

2nd step puts pivot in left location

3 4 2 1 5 $\boxed{5}$ 9 8 7

left list

right list

Quicksort worst case analysis

$$T(n) = T(i) + T(n-i-1) + cn$$

$i = |S_1|$ the no. of elements in one side of the array

Solve $T(n)$

$K=0$

$T(n) = cn + T(n-1)$

(with $i=1$ in the worst case)

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$K=1$

$= cn + \left( c(n-1) + T(n-2) \right)$

$= cn + cn - c + T(n-2)$

$K=2$

$$= cn + cn - c + (c(n-2) + T(n-3)))$$

$$= cn + cn + cn - c - 2c + T(n-3))$$

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$K=3$

$$= cn + cn + cn - c - 2c + (c(n-3) + T(n-4)))$$

$$= 4cn - c\sum_{K=0}^{n} K + T(n-4))$$

$$= c(K+1)n - \ldots + T(n-K+1)$$

$$\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$$

$$\begin{array}{l} 0+1+2+\cdots+n \\ n+(n-1)+\cdots \quad 0 \\ \hline n+n+\cdots+n \end{array}$$

$$= \frac{n(n+1)}{2}$$

$$T(n) = c(k+1)n - c\frac{n(n+1)}{2} + T(n-k+1)$$

let $k=n$

$$T(n) = c(n^2 + n) -$$

$$\frac{cn^2}{2} - \frac{cn}{2} + T(1)$$

$$\frac{cn^2}{2} - \frac{cn}{2} \in O(n^2)$$