

Midterm:

min: 15.5

max: 97.5

avg: 65.2

Q1.

Copy const: creates new object

Copy assign. op: operator = (==)

— gives to existing object

Q2

$p3 = p1 < p2 ? p1 : p2;$

if ( $p1.operator < (p2)$ )

$p3.operator = (p1);$

else

$p3.operator = (p2);$

Q3

Const-iterator } accessor

↑ derived

iterator } accessor  
+  
mutator

(

derived class expands

functionality of parent  
class

Q4 OOP

a) info hiding:

restrict access to

(private) object components  
(data)

b) encapsulation:

grouping of data &  
methods

in one application  
(i.e. class)

(i.e. class)

c) template: provide  
type-independence  
(generic programming)

d) inheritance:  
allows extension (or  
restriction) of a class  
(by deriving a class)

e) polymorphism:  
(e.g., overloading)

Q5

$$T(n) = \Omega(G(n)) \quad | \quad \exists c, n_0 :$$

$$T(n) \geq c G(n), \quad \forall n \geq n_0$$

"big-omega" : lower bound

(best case)

Q6

$\Theta(T(u))$ ,



average  
running time

$\Omega(T(u))$



transitional  
minimum  
running time

$$\text{if } (\Theta(T(u)) = \underline{\Omega(T(u))})$$

alg. runs optimally on average

$$\text{if } (\Theta(T(u)) = \Omega(T(u)))$$

alg. is optimal



Q7

Alg. 1:  $\Theta(n) < \Omega(n \lg n)$

on avg. better than insertion

→ impossible; no example

---

Alg. 2:  $\Theta(n^2) = O(n^2) > \Omega$

worst case = avg. case

ex. bubble sort

Alg. 3  $\Omega = \Theta = O$

optimal alg.

e.g. AVL, R-B, balanced tree alg.

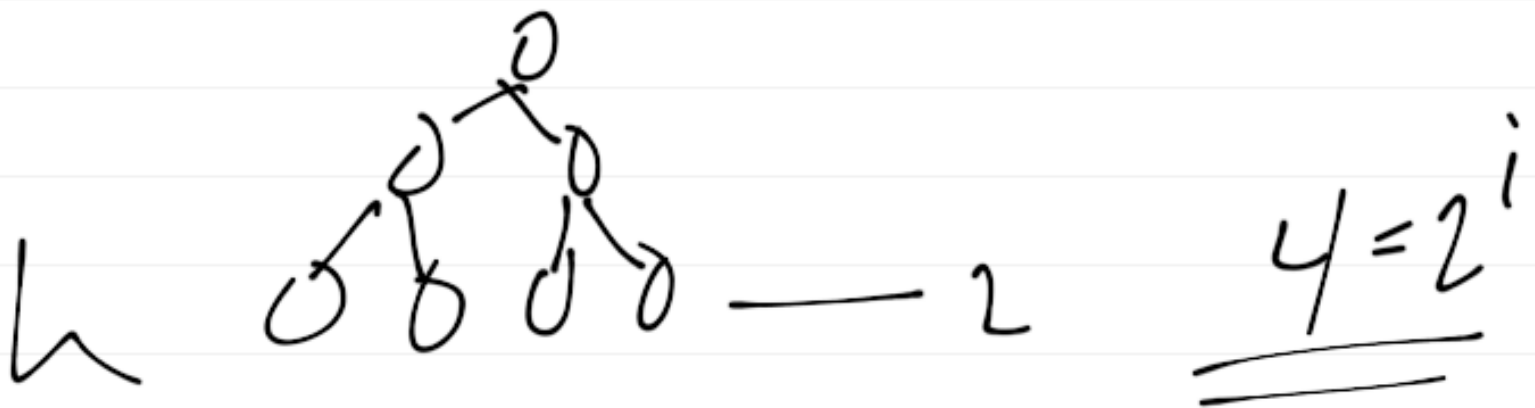
Alg. 4  $\Omega = \Theta = n \lg n$   
 $O(n^2)$

depenent on worst case

e.g., binary search tree

Q8

level  $\emptyset$  # rows  $1 = 2^i$



$2^i$  nodes per level

$$\sum_{i=0}^h 2^i = \boxed{2^{h+1} - 1}$$

# leaves:  $2^h$

Q9 Heap insert:  $O(\lg n)$

ST 1. A



slightly better

$$O(\lg n) \times n = O(n \lg n)$$

for insert  
+

$$O(\lg n) \times n = O(n \lg n)$$

for Delete Min

---

$$O(2n \lg n) \in O(n \lg n)$$

$$O(1) \times n = O(n)$$

for Top

+

$$O(n) \times 1 = O(n)$$

Fix heap

+

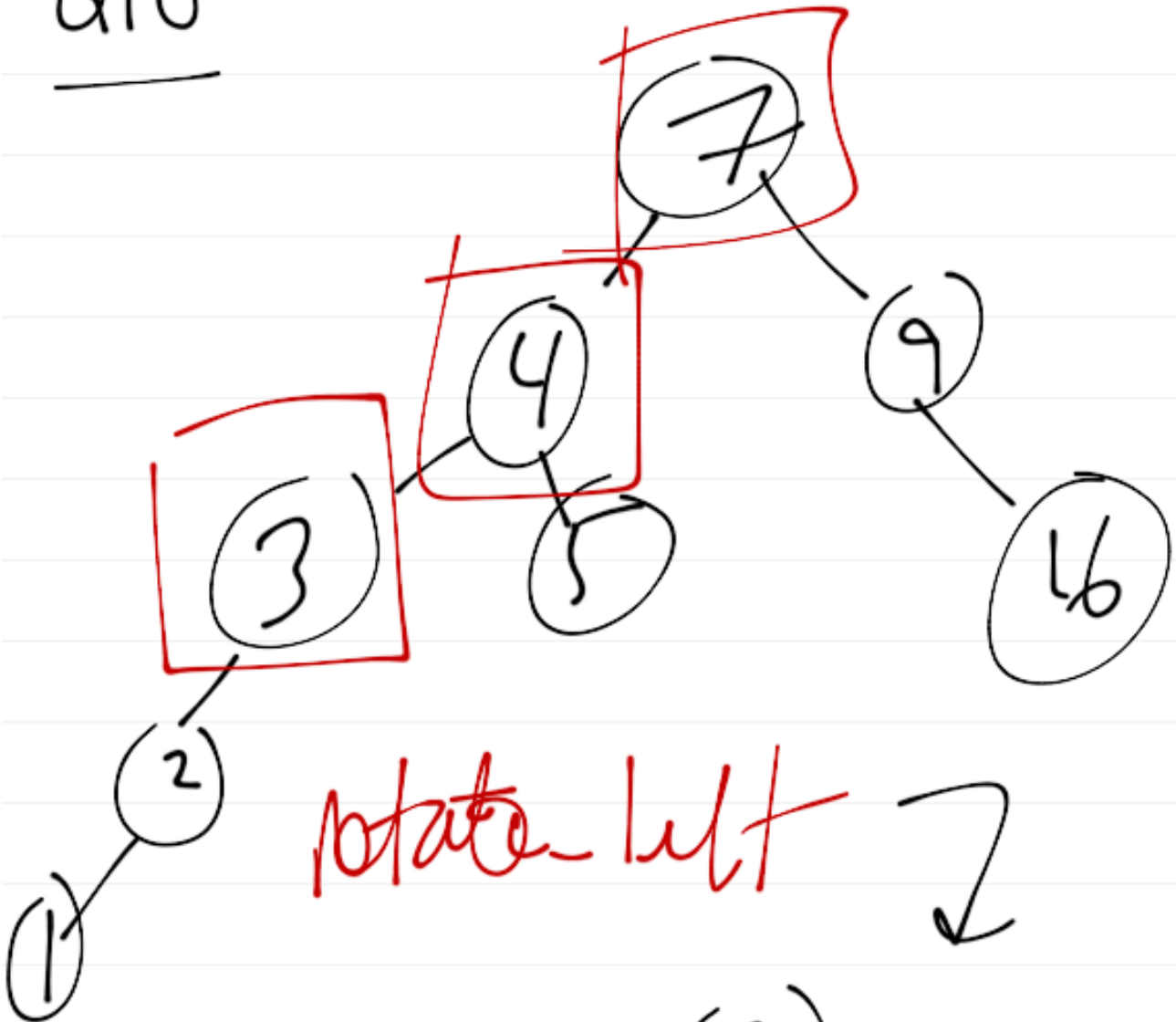
$$O(n \lg n) \text{ Delete}$$

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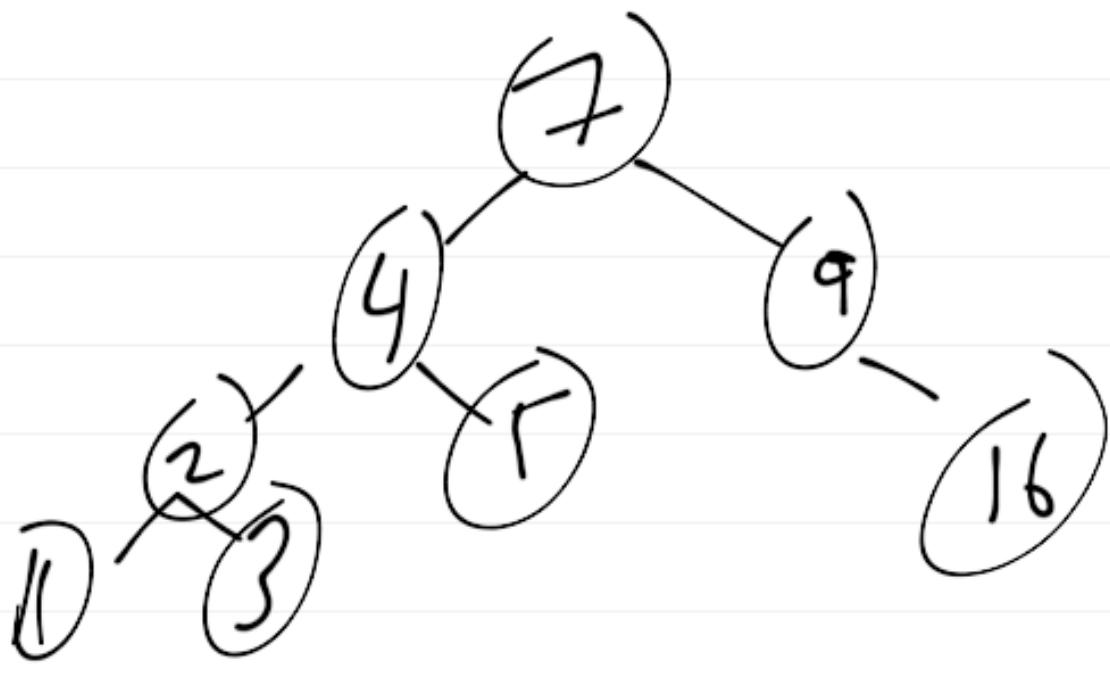
$$O(n \lg n + 2n)$$

$$\in O(n \lg n)$$

Q10



rotate-left ↴



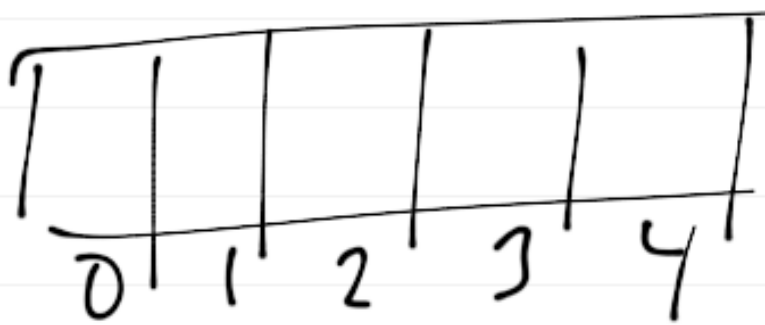
# Hashing (Chp. 5)

- data structure is a hash table
- basic array that supports  $O(1)$  insertion, deletion, find  
(avg. constant time)

- Given Key (data element, e.g. a number, a string)

Compute its index into the array via hash function

e.g. a very small hash table:



hash table of size 5 ( $n=5$ )  
given any number, e.g.,  
19203, 32762, etc.,

simple hash function is

$$\text{Number} \bmod n \quad n = \text{size}$$



any number  $\geq 5$   
will give index  $\in [0, 4]$

— two major concerns:

1. if no. elements  $\gg n$ ,  
we'll get collisions,

how to store:



2. Performance of Mark  
function, e.g., given  
 $i \in \mathbb{Z}_5$ , let's say  
all numbers are multiple  
of 5, 5, 10, 15, 20, ...  
 $\Rightarrow$  all go in index  $[i]$   
(unbalanced table)

- a good candidate for  
hash table size & hash  
function is the prime numbers

1, 3, 5, 7, 11, 13, 17,

19, 23, 29, 31, 37,

41, 43, 47, 53, 59,

... 10, 007

- text gives good hash

function for string

keys

$$h = \sum_{i=0}^{\text{keysize}-1} \text{key}[\text{keysize}-i-1] \times 37^i$$

via Horner's rule:

$$h_n = k_0 + 37k_1 + 37^2 k_2 =$$

$$((k_2)37 + k_1)37 + k_0$$

e.r  
 $h = 0$   $(i++)$

for (int i = 0; i < key.length();

$h = 37 * h + key[i];$

$h = h \% tableSize$

if ( $h < 0$ )  $h += tableSize$

- implementing the hash table (doesn't seem to exist in C++ STL)

```
class hash_t
```

```
{
```

```
private:
```

```
std::vector< std::list<T>>
```

```
elements;
```

```
}
```

- Analysis is tricky:
  - need to consider average case  $O()$

ex. f.

$$O(1) + O\left(\frac{n}{\text{size}}\right)$$

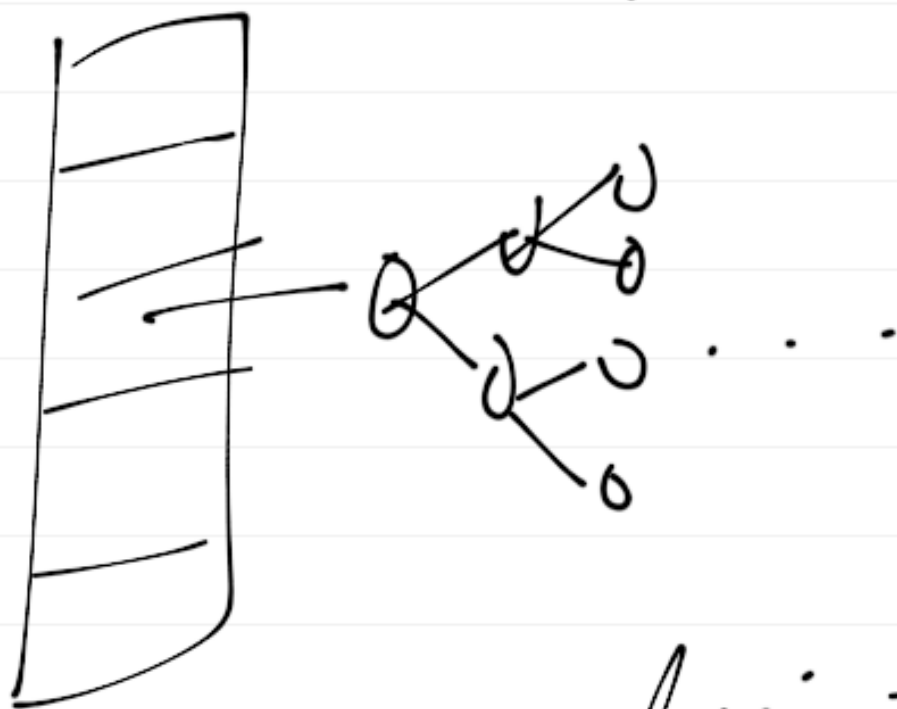
hash fun                      search



- memo do better.

- hash table of  
hash tables

- hash table of trees



- average case analysis  $\equiv$   
amortized analysis