Eye tracking in Algebra

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ABSTRACT

Without a doubt the concepts of variables and functions are the cornerstone of all mathematics as we learn them today. To an open mind, most problems in the world may have a possible way to be solved or optimized. We want to discuss the students/people who have had a fear of mathematics and the importance of how they look at variables in general. This problem goes a bit deeper than simple algebra. Many students, in our experience refuse to even attempt logarithmic or trigonometric functions because they do not know what they are looking at. We will table this issue for now and narrow the scope of our research to "variables vs shapes". Using eye tracking equipment, we record the point of regard of a random group of peers performing an inspection task in a virtual reality simulator. Analysis of their eye movements leads to a visualization of their scan-paths and allows us to display the students visual search strategy. Furthermore, we show and discuss the differences in these scan path patterns.

KEYWORDS

Eye tracking, Algebra, Cognitive process

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1 INTRODUCTION

Algebraic and geometric problems form part of the core of arithmetic required in primary education for people to learn. Algebra is necessary for people to understand how to solve for unknown variables, while geometry is necessary for the visualization and measurement of physical forms. Each type of problem also asks for a different approach and understanding, such as the necessity of understanding the equals and negative signs in algebra [2]. Geometry, on the other hand, requires a deduction of the given elements in the problem based on known laws of Euclidean geometry [11]. Both of these methods require critical thinking, as well as some degree of visualization. While the learning methods of each of these bases of arithmetic have been well explored, knowledge of how individuals go about solving them from a visual standpoint have not. When a person reads, their eyes follow the text in front of them in the direction it is meant to be read. An algebraic equation or geometrical unknown, on the other hand, may not provide such a linear method of comprehension for the person solving the problem. The eyes of the person solving the problem may go all over the page; they may go back to the same variable several times; they may even stay on one point for an extended period of time if the user is attempting to focus on a particular aspect of the problem. These things are unknown as of now, as few if any studies have been done to explore this subject of eye tracking. Therefore, it is the purpose of this study to explore the eye movements of participants in order to understand how people solve mathematical problems by looking at them.

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2 BACKGROUND

Eye tracking studies have an important assumption that people's attention focuses on what they are looking at [12]. Eye trackers record human's visual attention into objective and quantitative data, which reveal human visual attention during information perception process and cognitive process [3]. Traditional research methods like interviews and think-aloud sessions have some problems that may threat the internal validity of the study of students' cognitive process during equation solving. For example, interviews may yield subjective results according to different participants and think-aloud sessions have influence on the information processing process during the experiment [16].

Meanwhile, in the last decades, the application of eye tracking based researches have shown its promising power in the field of neuroscience, psychology, computer science, human factors research and cognitive process researches such as reading [3, 4, 10, 13, 14, 16]. Eye tracking methods also have good adaption to the research related to online search [8], information processing research [13, 14] and educational research [16, 17]. Eye tracking technology records the eye movements when the participant is looking at certain content [1, 4]. According to Ehmke & Wilson's (2007) definition, sample metrics of eye tracking data include: 1. the fixation, when the participant's eye stare at the same position of the screen; 2. A saccade or scan path, when the participant move eyes from one position to another position; and 3. "A gaze plot may be used to show the succession of fixations and saccades on a screen or webpage for an individual user, while heat maps show how long each part of a screen has been looked at." [4] Particularly for this study, the fixation duration and gaze plot may be powerful to provide critical information to reveal the participants' strategy and cognitive process in solving algebra problems.

The application of eye-tracking technique on cognitive and learning process have many successful examples. A study suggests that when solving multiple choice questions on a computer screen, successful problem solvers tend to focus their visual attention more on the relevant factors while failures have troubles in decomposing the problem and figure out the relevant cues [17]. Another study suggests that the students' strategy in solving equations are impacted by the types of equations and their performance are getting better along with the time of the experiment goes on [16]. Numerical representation and spatial representation are believed to have close cognitive link in the field of mathematics [7, 18]. Visual representations in mathematics have serious impact on the understanding of math equations and problems [5]. Eye tracking technique can track the real-time mental process by linking the visual spatial attention to their gaze [6]. Also, the link between visual attention and numerical numbers could reveal the mental attention and information processing process in activities like mental arithmetic [15].

As online education is believed as "one of the fastest growing trends in the educational techniques" [9] by the U.S. Department of Education, most U.S. students may have already gotten experience of learning or trying to solve algebra equations online: that is, by seeing equations on a computer monitor to learn them. Eye tracking is recommended as a suitable and promising technique to study the cognitive process of online learning [17]. Thus, this study is also intended to serve as a case study in examining the cognitive process of students in online learning of algebra, through varying different components of the algebra equations to explore the equation solving process and learning process of the students.

Our hypotheses are that: 1, the variation (different symbols, shapes and colors, etc.) of algebra equation symbols may impact the speed of students' cognitive process; 2, the variation (different symbols, shapes and colors, etc.) of algebra equation symbols may impact the participants perceived difficulty of the equations and 3) the variation (different symbols, shapes and colors, etc.) of algebra equation symbols may impact the error rate of the participants when solving the equations.

3 METHODOLOGY

3.1 Apparatus

The process of our research uses the GazePoint eye tracking system to collect ocular movement during HCI tasks. The system is non-contact and captures eye and head movement with camera-based tracking. For the current study the system was configured for precision gaze using infrared target tracking. The sampling rate was 60 Hz and the typical gaze position accuracy less than 0.5'1' rotational error. A Dell 22' monitor with a (1920 x 1080) resolution was used. The participants will be seated at a distance of 24 inches from the monitor.

3.2 Participants

We plan to use participants in the Clemson School of Computing. Ideally we will be able to sample 20 students for variables and 20 students for shapes. The impact of using college level students proposes a few variables worth discussing. Each participant will be from age 18-35 with mathematics being a relatively prevailing school of thought for each person. This study would be more well rounded for young students taking algebra for the first time but this pool of participants will still generate usable data pertaining to our hypothesis.

3.3 Stimuli

There are 2 cases the subject could experience. A cardinality 3 system of simple equations; both with the same semantics but different symbols. This will be a simple system using only addition and multiplication with integers ranging from 0-9, designed to be a slight challenge with the meager time interval. For the variable case, the system of equals will be limited to variable representations as 'x' and 'y'. While the shape case will utilize colored triangles and rectangles. Additionally, we will include a white screen with a central black dot as a starting position for the participants eyes. Finally, we will also provide an answer box for participants to record their calculation and their perceived difficulty of the computation. Once all data is gathered the monitor will return to a black screen.

3.4 Design

The experiment will take advantage of 2 randomly allocated groups of equal size. Independent measures will be taken from a control group and an experimental group. The experimental group solving with colored shapes while the control solves using traditional variables. Figure 1 and figure 2 below are two examples of the (shape and symbol) equations that we are using.

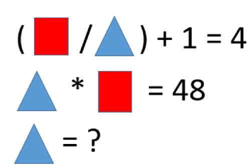

Figure 1: Sample shape equation

$$2(A + B) = 16$$
$$2(B - A) = 12$$
$$A = ?$$

Figure 2: Sample symbol equation

Furthermore, each group will end each case by recording their supposed calculation along with an ordinal one question survey of their experience with the experiment.

3.5 Procedure

When executing our experiment, the participant will be read the purpose of our study and prepare to be guided through the steps of synchronizing the eye tracking software. First, they will sit 24 inches from the monitor. They will then be read a small paragraph describing the legal ramifications and risks involved with participating in the study. Once gathering consent from the participant will complete a 9-point visual synchronization. After synchronized the subject will be looking at a blank screen until the test has started. One of the 2 cases the subject could experience will appear on the screen. A cardinality 3 system of simple equations. One group of test subjects will have 15 seconds to solve for '?' using a system of equations represented with variables. While the other group will be asked to solve the same system represented with shapes. After the subjects' eye path is recorded, the system of equations is removed and they will be promoted with a box to record their supposed result. Once a participant has recorded his/her calculation they will be promoted with a subjective question about the difficulty of the system of equations; from 1-5, 1 being easy and 5 being impossible. After the subject responds with their opinion the experiment will be over and they will be allowed to leave if they have no questions.

3.6 Data Plan

With a goal of 24 participants in all we strive to find a meaningful difference in styles of occur assessment between variables and shapes. As evidence of this claim we will process a number of different comparisons. First and foremost to calculate a feel for a difference in results we will compile the total number of correct and incorrect calculation for each category. Furthermore, we will calculate the average number of vertical fiction changes recorded for each category. This will be accomplished by capturing 3 rectangular fixation regions representing each line of the equations. These two calculations will show insight into the mind of students trying to process a variable rather than a shape. Along with the opinion based calculations on the difficulty of the matching equations we will be able to infer tangible differences in approaches. In an attempt to derive the cause of miscalculation, we will compare and contrast correct and incorrect focus maps for both categories. Although only a simple experiment we hope to gather insight on the primary mechanics of horizontal ocular analysis and the effects of variable on its efficiency. We aim to not to discover a fundamental lack of ability in some students but to merely propose their own fear of variables is holding them back; not only in mathematics but in everyday life as well.

4 RESULTS

4.1 Descriptive statistics

All the data analysis is done by using R version 3.5.1. Microsoft Excel and SPSS 24.0 are used to store and handle data in basis. This study is approved by Clemson University Institutional Review Board (IRB) (IRB number: IRB2018-349) before any data collection.

14 participants are recruited for this experiment. All the participants are Clemson University students, most of them are undergraduate students. 35.71% of them are female (5 out of 14) and 64.29% of them are male (9 out of 14). The average age is 23.14 (SD=2.35) years old (see figure 3 below). After the experiment, we asked the participants about their perceived difficulty of the two different types of equations. 70.83% (17 out of 24) of our participants perceived that using shapes to represent the unknown values in equations is more comfortable and contributed to their understanding of the questions.

4.2 Questionnaire and eye-tracking data results

Two equations are using shapes to represent the unknown values, and the other two equations are using symbols to represent the unknown values. From our data, the error rate of solving each

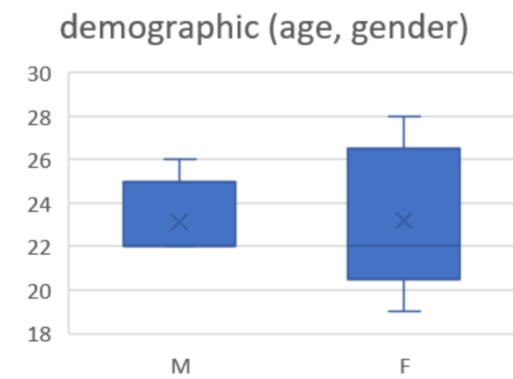

Figure 3: boxplot of participants' age distribution by gender

type of equations is calculated. If the equation never gets a correct answer from the participants within the time limit or the participants give up, it is marked as âĂIJwrongâĂİ and the participants skip to the next equation. The participantsâĂŹ overall error rate of solving equations using shape is 28.57%. In the same time, the participantsâĂŹ overall error rate of solving equation using symbol is also 28.57%. The error rate is exactly the same for solving both types of equations, which surprise the researchers. One potential explanation is coincidence, and another potential explanation is that the pilot study (which we did to ensure the two types of equations are of the same level of difficulty.) is quite successful.

Figure 4 below shows the boxplot of time used to solve each equation. If the equation is not solved within the time limit, the time is recorded as 60 seconds. Some participants experienced difficulty in solving equation 2, which makes the upper bound of time in solving equation 2 a little bit high. The descriptive statistics and ANOVA table of time used to solve each equation is shown in Figure 3 and Figure 4. Equation 1 and equation 3 used shapes to represent unknown values and equation 2 and equation 4 use symbol to represent unknown values.

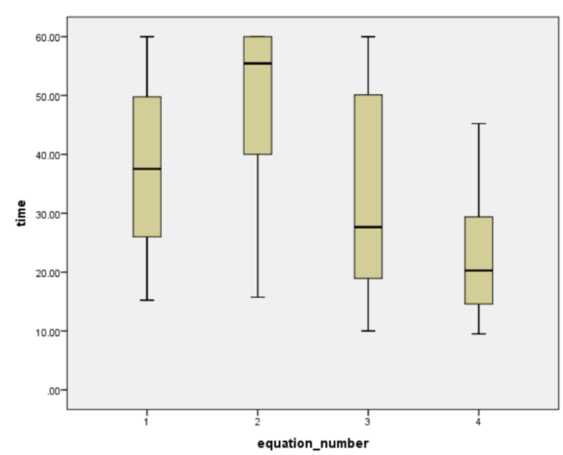

Figure 4: boxplot of time to solve each equation

Descriptives

time

		N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum	Between-Component Variance
1		14	38.1971	15.10539	4.03709	29.4756	46.9187	15.24	60.00	
2		14	49.2543	13.92054	3.72042	41.2168	57.2918	15.76	60.00	
3		14	32.2429	16.74361	4.47492	22.5754	41.9103	10.00	60.00	
4		14	23.1079	11.01645	2.94427	16.7471	29.4686	9.50	45.24	
Total		56	35.7005	16.92454	2.26164	31.1681	40.2330	9.50	60.00	
Model	Fixed Effects			14.34978	1.91757	31.8526	39.5484			
	Random Effects				5.48079	18.2582	53.1428			105.44779

Figure 5: Descriptives of time to solve each equation

ANOVA

time

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	5046.556	3	1682.185	8.169	.000
Within Groups	10707.645	52	205.916		
Total	15754.200	55			

Figure 6: ANOVA table of time to solve each equation

Descriptives

time

		N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean Lower Bound	95% Confidence Interval for Mean Upper Bound	Minimum	Maximum	Between-Component Variance
1		28	35.2200	15.93848	3.01209	29.0397	41.4003	10.00	60.00	
2		28	36.1811	18.13766	3.42770	29.1480	43.2141	9.50	60.00	
Total		56	35.7005	16.92454	2.26164	31.1681	40.2330	9.50	60.00	
Model	Fixed Effects			17.07352	2.28154	31.1263	40.2748			
	Random Effects				2.28154[a]	6.7108[a]	64.6903[a]			-9.94906

a. Warning: Between-component variance is negative. It was replaced by 0.0 in computing this random effects measure.

Figure 7: Descriptives of time to solve each type (shape and symbol) of equation

ANOVA

time

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	12.931	1	12.931	.044	.834
Within Groups	15741.269	54	291.505		
Total	15754.200	55			

Figure 8: ANOVA table of time to solve each type (shape and symbol) of equation

From figure 7 and 8, we can see the results for each type (shape and symbol) of equation is non-significant (p=0.834). The average time used to solve each type (shape and symbol) of equation is 35.22 seconds and 36.18 seconds respectively. This may be due to the amount of equations used is not large enough, more equations in each type may yield more accurate results but also will increase the time for the experiment. The sample size is also a limitation of this study. The boxplot of average time to solve each type of equation is shown in Figure 9 below.

Figure 8-10 shows some samples of the fixations and heat map with the stimulus equation. The average number of fixations for each type of equations is also measured.

Overall, for the equation with shapes, the average number of fixation is 63 in the AOI in the equations, the average total fixation

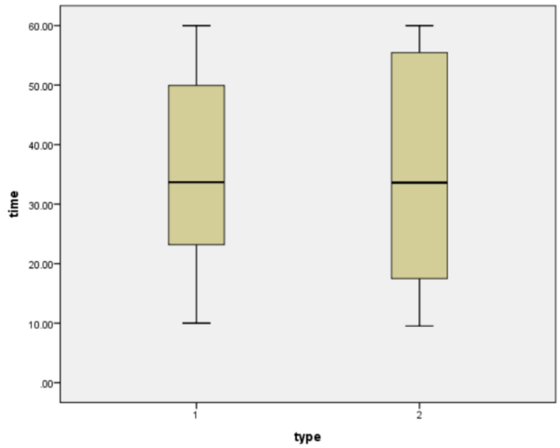

Figure 9: boxplot of time to solve each type (shape and symbol) of equation

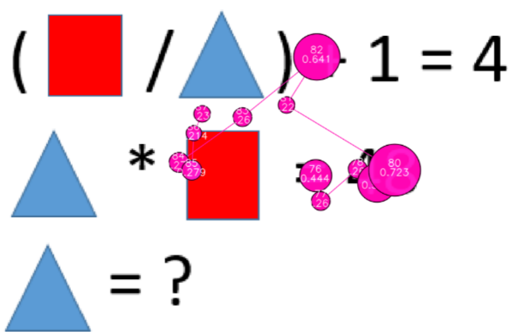

Figure 10: sample fixation in the equations

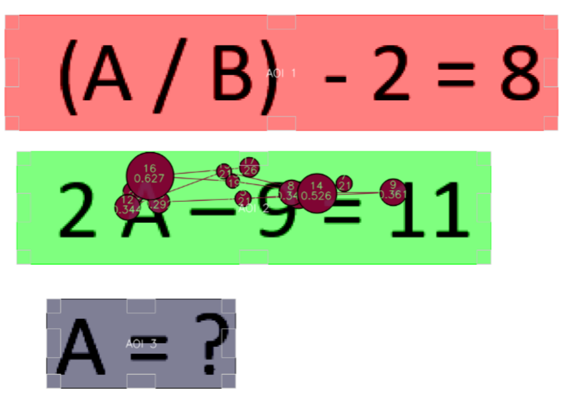

Figure 11: sample fixation with Area of Interest (AOI) (the three colored rectangles)

duration in the AOI is 29.55 seconds. For the equation with symbol, the average number of fixation is 92 in AOI of the equations, with the average fixation duration is 33.31 seconds. the equations

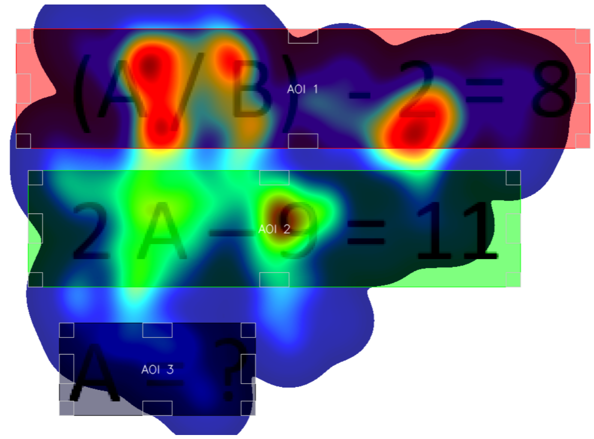

Figure 12: sample heat map of one equation

with symbols generally require a little bit more fixations than that with shapes. This may be an explanation to why the participants perceived shapes are better for their understanding of the equation.

5 DISCUSSION

In the original hypothesis we predicted that some students have a mental block when it comes to learning various forms of mathematics. Throughout the data collection we ran into quite a few road blocks. Some equations we wanted to use were much too hard for Clemson Undergrad students, whereas some students had no difficulty at all completing even the hardest equations we could generate. Although we were not able to analyze any of the data from these early stages, we believe these trials to be worth discussing based on the merit of the issues with their design and the results they yielded. For example, many students saw the higher level equations and simply admitted "It is too hard for me". This lack of trying could be a vestige of the mental block in students we are trying to pin down.

Furthermore, the data we were able to extrapolate shows that the Shape equations were either solved extremely fast and correct or the subject maxed out the 60 second time limit. This hit-or-miss pattern for shapes is very interesting when evaluating the validity of the claim. We see evidence of the reason we began researching this topic. Why would a participant not be able to solve the equation with shapes? Especially when they were able to solve the equations of the same difficulty using variables. We answer these equations by simply relying on the hypothesis. The students who answered the shape questions extremely fast and correct were able to see past the shapes and see the pure logic of the equations. The students who did not answer simply spent too much time juggling the syntax of the equations diluting the workability of the logic. These results for shape equations are especially interesting because we subtle differences in the variable equation data.

What we mean by this is, variables had a much wider variance of completion times. According to our post experiment interviews; the issue of syntax juggling was not found when solving the variable equations. This means participants who either successfully or unsuccessfully solved the equations was mutually exclusive from our hypothesis. A lack of a mental block for variable equations is indicative of our American mathematics education. In our learning process, American students often on ever work with the variables X and Y. This means many students are uncomfortable using anything other than X or Y. These distinct differences from the results of the shape and variable serve our discussion well when deriving the validity of our hypothesis.

We believe these results, along with the accompanying field interviews are the strongest support we were able to generate. More clearly, the wide variety of completion times and correct answers lead us to believe some students are able to overcome uncertainty, while others folded to their own brain as discussed previously. Bringing numberer into our discussion, the subjects showed an error rate of around 28 percent for both shapes and variable equation. To validate our claim of syntax juggling, we conducted a series of post experiment interviews. Of the 28 percent of students who had an error computing a shape equation, 70 percent found themselves running out of time. When questioned the average response was "I spent most of my time translating the shapes into variables". These emotions responses from students unable to do mental math on command became a large supporting factor in our research.

Unfortunately, we didn't reach our goal number of participants which brings doubt to our claims. With only 14 usable data sets and interviews our empirical analysis is traditionally lackluster. This was due to malfunctions in the data collection of a handful of participants. Participants with corrupted data were not given the opportunity to be interviewed or rerun through the experiment. Nevertheless, we believe our small data set has at least some evidence to support our hypothesis. We do not claim this to be proof of anything. But for the sake of discussion we ask "If these meant blocks are real, how can we over come them and where do they come from". One group of researchers alone do not have the tools to answer these questions. Hopefully our research has at least piqued the interest of the scientific community in a way that leads to answers to these questions.

6 CONCLUSION

The analysis we have presented is a mere drop in the bucket of understanding cognition and learning. With so many unanswered questions, this area of research is in desperate need of more, valid, data. As eye tracking hardware becomes more sophisticated and inexpensive it is our hope that students will be able to utilizes and review their own eye tracking data. Being able to see how the teacher's eyes pieced together a series of equations, or how their eyes debug code. This the future we see for education in America. At the end of the day, all of our analysis could be seen as supporting evidence for our claim or as a reminder that everyone is different and acts differently. The lines of which is has become blurred when evaluating the true nature of people. Especially with only a handful of datasets to work from.

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