Robotics 2
Camera Calibration

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What is Camera Calibration?

- A camera projects 3D world-points onto the 2D image plane
- **Calibration**: Finding the quantities internal to the camera that affect this imaging process
  - Image center
  - Focal length
  - Lens distortion parameters

Motivation

- Camera production errors
- Cheap lenses

- Precise calibration is required for
  - 3D interpretation of images
  - Reconstruction of world models
  - Robot interaction with the world (Hand-eye coordination)

Projective Geometry

- Extension of Euclidean coordinates towards points at infinity

$$\mathbb{R}^n \to \mathbb{P}^n : (x_1, \ldots, x_n) \to (\lambda x_1, \ldots, \lambda x_n, \lambda) \in \mathbb{R}^{n+1} \setminus \mathbf{0}_{n+1}$$

- Here, equivalence is defined up to scale: $\hat{\mathbf{x}} \sim \hat{\mathbf{y}} \Leftrightarrow \exists \lambda \in \mathbb{R} \setminus \{0\} : \hat{\mathbf{x}} = \lambda \hat{\mathbf{y}}$
- Special case: Projective Plane $\mathbb{P}^2$
- A linear transformation within $\mathbb{P}^2$ is called a Homography

Homography

$$\begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} \sim \underbrace{\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix}}_{\text{Homography } \mathbf{H}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

- $\mathbf{H}$ has 9-1(scale invariance)=8 DoF
- A pair of points gives us 2 equations
- Therefore, we need at least 4 point correspondences for calculating a Homography

Pinhole Camera Model

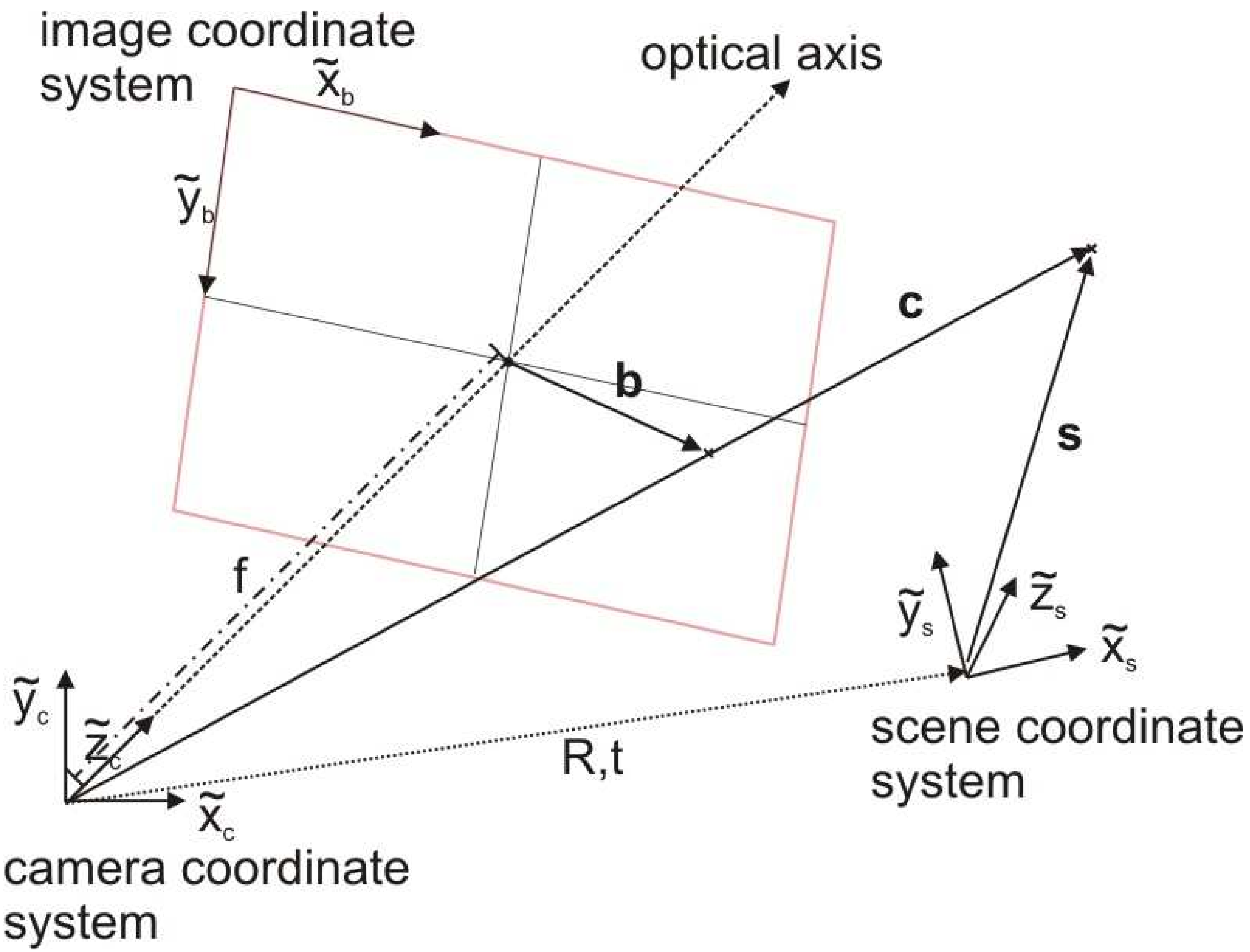
- Perspective transformation using homogeneous coordinates:

$$\mathbf{b} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Intrinsic
camera parameters

Extrinsic
camera parameters

Pinhole Camera Model



Pinhole Camera Model

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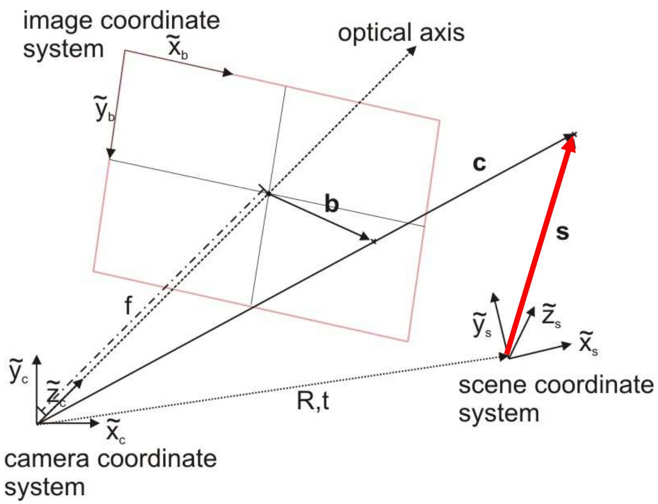

world/scene coordinate system

Pinhole Camera Model

- Perspective transformation using homogeneous coordinates:

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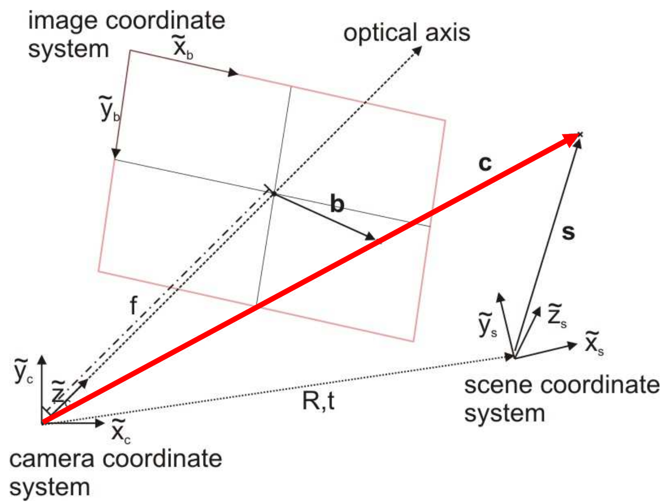

camera coordinate system

Pinhole Camera Model

- Perspective transformation using homogeneous coordinates:

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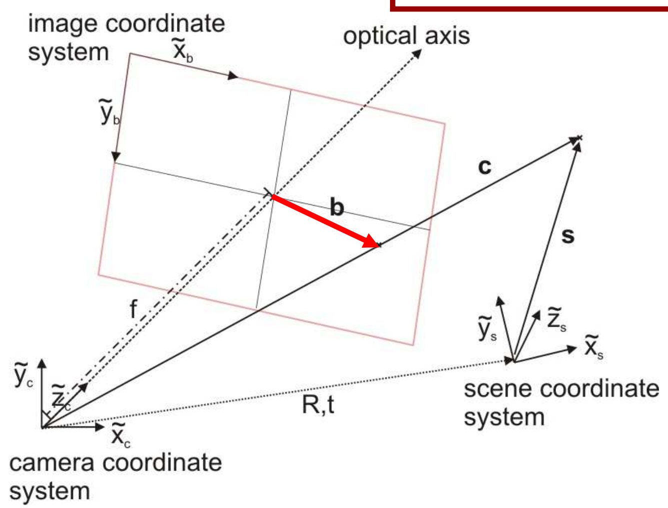
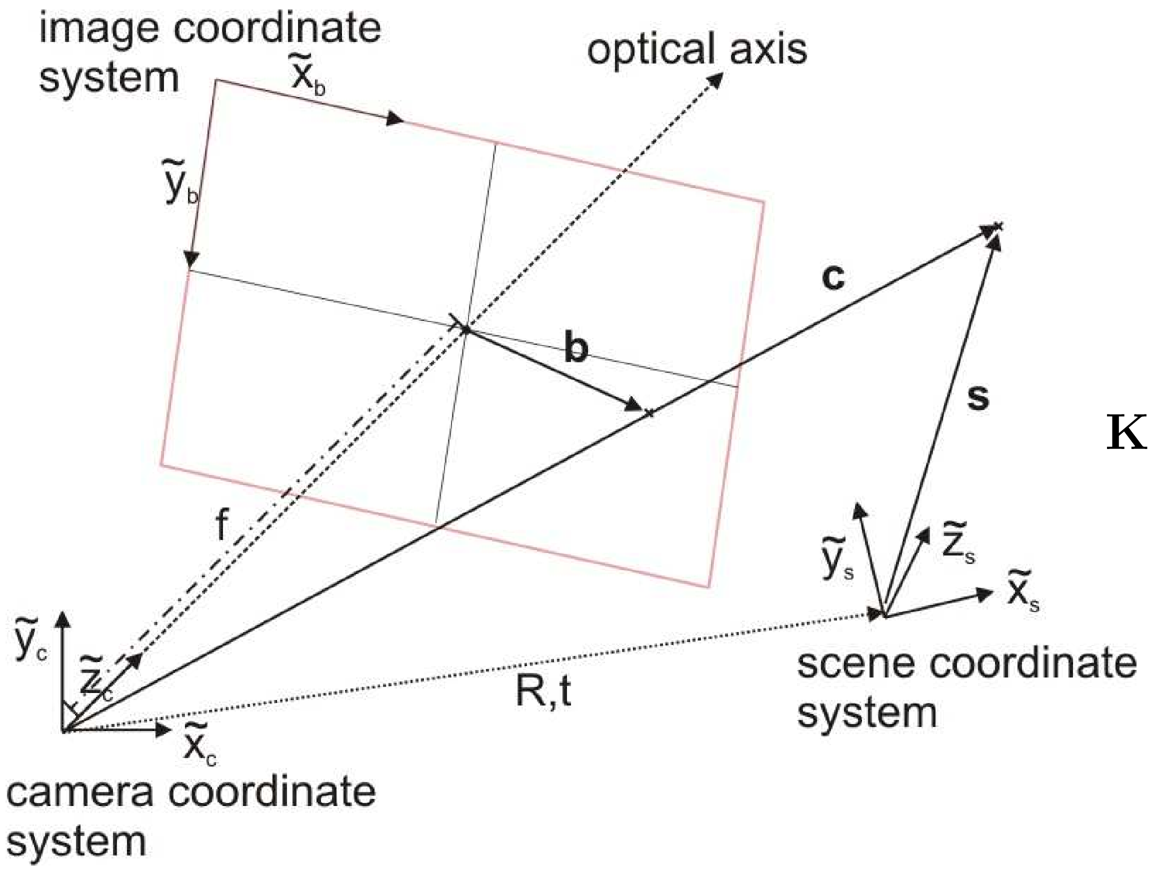


image coordinate system

Pinhole Camera Model

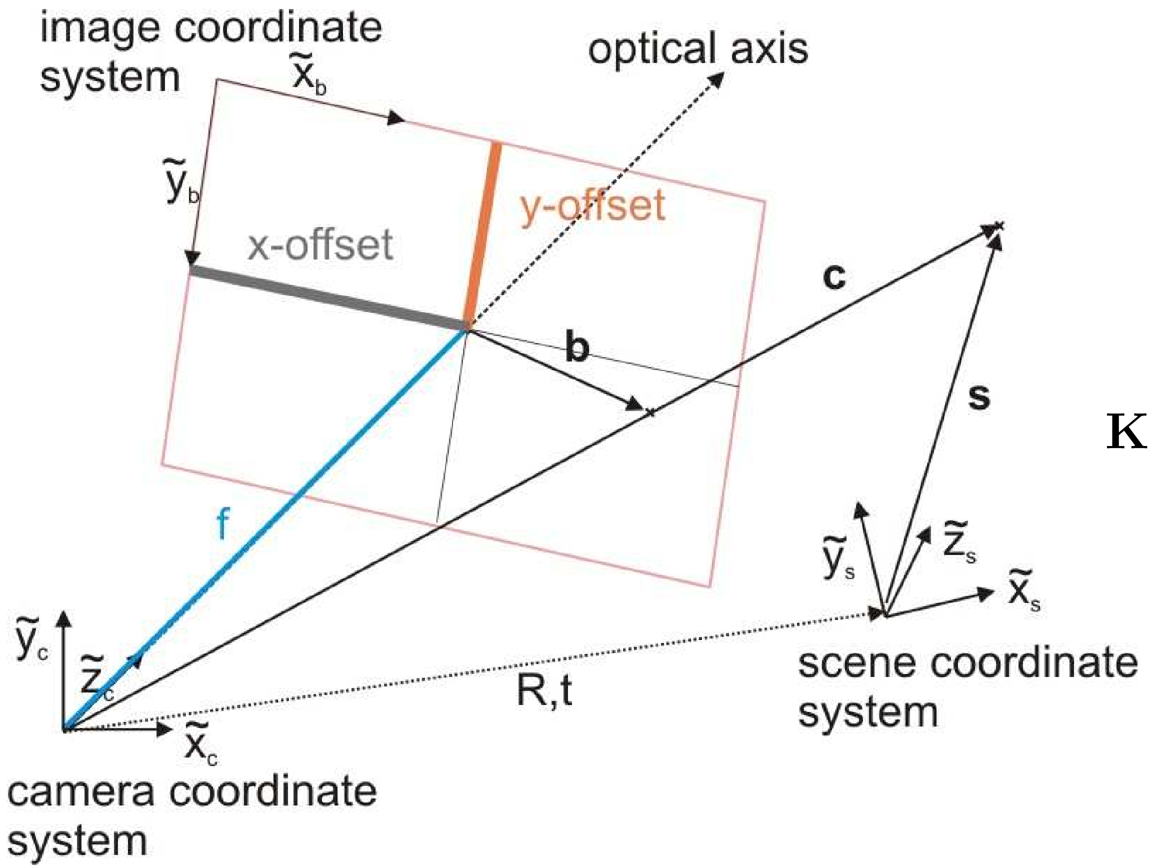
- Interpretation of intrinsic camera parameters:



$$\mathbf{K} = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

Pinhole Camera Model

- Interpretation of intrinsic camera parameters:

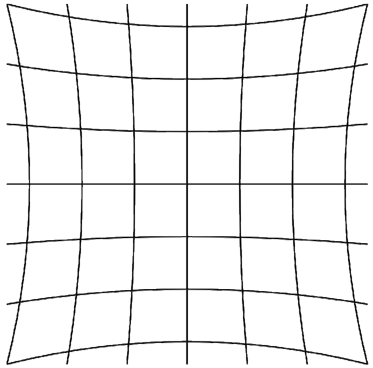


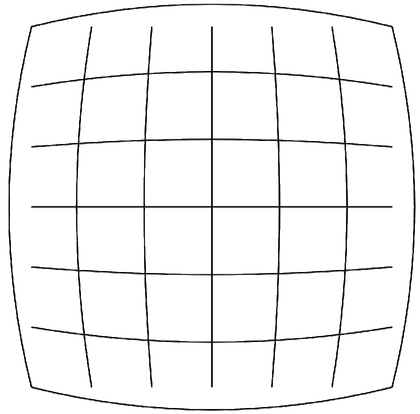
focal length
x-offset

$$K = \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

y-offset

Lens distortion





Non-linear effects:

- Radial distortion
- Tangential distortion

- Compute corrected image point:

(1) $$x' = x/z$$
$$y' = y/z$$

(2) $$x'' = x'(1 + k_1 r^2 + k_2 r^4) + 2 p_1 x' y' + p_2 (r^2 + 2 x'^2)$$
$$y'' = y'(1 + k_1 r^2 + k_2 r^4) + p_1 (r^2 + 2 y'^2) + 2 p_2 x' y'$$

where $r^2 = x'^2 + y'^2$

k_1, k_2 : radial distortion coefficients

p_1, p_2 : tangential distortion coefficients

(3) $$u = f_x \cdot x'' + c_x$$
$$v = f_y \cdot y'' + c_y$$

Camera Calibration

- Calculate intrinsic parameters and lens distortion from a series of images
  - 2D camera calibration
  - 3D camera calibration
  - Self calibration

Camera Calibration

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need external pattern

Camera Calibration

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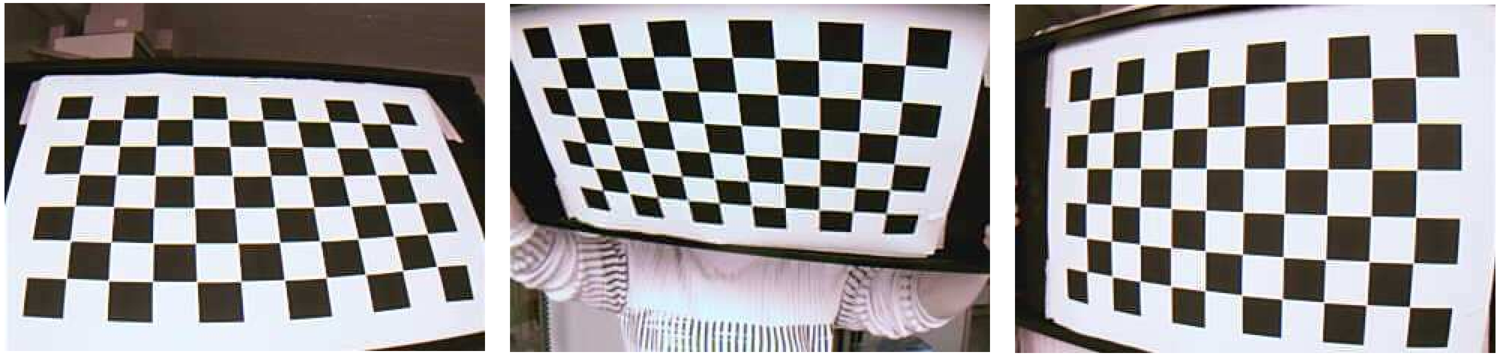
2D Camera Calibration

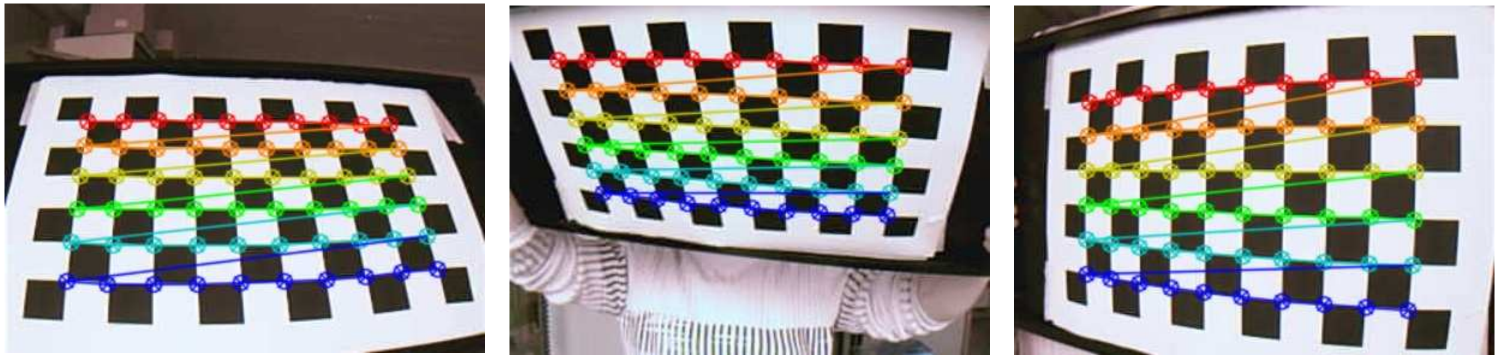
- Use a 2D pattern (e.g., a checkerboard)



- Trick: set the world coordinate system to the corner of the checkerboard

2D Camera Calibration

- Use a 2D pattern (e.g., a checkerboard)



- Trick: set the world coordinate system to the corner of the checkerboard
- Now: All points on the checkerboard lie in one plane!

2D Camera Calibration

$$\mathbf{b} = \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} \sim \begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ \boxed{0} \\ 1 \end{pmatrix}$$

- Since all points lie in a plane, their z component is 0 in world coordinates

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- Since all points lie in a plane, their z component is 0 in world coordinates
- Thus, we can delete the 3rd column of the Extrinsic parameter matrix

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- Since all points lie in a plane, their z component is 0 in world coordinates
- Thus, we can delete the 3rd column of the Extrinsic parameter matrix

2D Camera Calibration

$$\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix}}_{(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})}$$

$$\Rightarrow \quad (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \mathbf{K}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})$$

2D Camera Calibration

$$\mathbf{H} = (\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \underbrace{\begin{pmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{pmatrix}}_{(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})}$$

$$(\mathbf{h}_1, \mathbf{h}_2, \mathbf{h}_3) = \mathbf{K}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{t})$$

$$\Rightarrow \quad \mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

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$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

- Note that $(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ form an orthonormal basis, thus: $\mathbf{r}_1^T \mathbf{r}_2 = 0, \quad \|r_1\| = \|r_2\| = 1$

2D Camera Calibration

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$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

$$\xrightarrow{\mathbf{r}_1^T\mathbf{r}_2 = 0} \quad \mathbf{h}_1^T\mathbf{K}^{-T}\mathbf{K}^{-1}\mathbf{h}_2 = 0$$

2D Camera Calibration

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$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$\|\mathbf{r}_1\| = \|\mathbf{r}_2\| = 1$ ⟹

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

2D Camera Calibration

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$$\mathbf{r}_1 = \mathbf{K}^{-1}\mathbf{h}_1, \qquad \mathbf{r}_2 = \mathbf{K}^{-1}\mathbf{h}_2$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 = \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2$$

$$\Rightarrow \quad \mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 - \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0$$

2D Camera Calibration

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2D Camera Calibration

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0 \qquad (1)$$

$$\mathbf{h}_1^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_1 - \mathbf{h}_2^T \mathbf{K}^{-T} \mathbf{K}^{-1} \mathbf{h}_2 = 0 \qquad (2)$$

- $\mathbf{B} := \mathbf{K}^{-T} \mathbf{K}^{-1}$ is symmetric and positive definite

2D Camera Calibration

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- $\mathrm{B} := \mathbf{K}^{-T} \mathbf{K}^{-1}$ is symmetric and positive definite
- Thus: $\mathrm{B} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{23} \\ b_{13} & b_{23} & b_{33} \end{pmatrix}$

Note: K **can be calculated from** B **using Cholesky factorization**

2D Camera Calibration

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- define: $\mathrm{b} = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33}) \qquad (3)$

2D Camera Calibration

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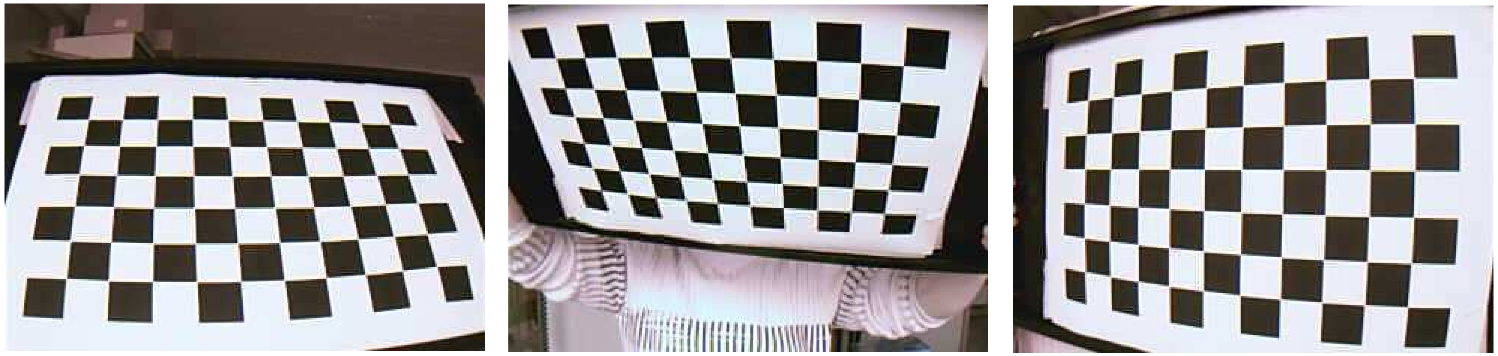
Note: K **can be calculated from** B **using Cholesky factorization**

- define: $\mathbf{b} = (b_{11}, b_{12}, b_{13}, b_{22}, b_{23}, b_{33}) \qquad (3)$
- Reordering of (1)-(3) leads to the system of the final equations: $\mathrm{Vb} = 0$

Direct Linear Transformation

- Each plane gives us two equations
- Since B has 6 degrees of freedom, we need at least 3 different views of a plane



- We need at least 4 points per plane

Direct Linear Transformation

- Real measurements are corrupted with noise

➡ Find a solution that minimizes the least-squares error

$$b = \arg\min_{b} \mathbf{V}\mathbf{b}$$

Non-Linear Optimization

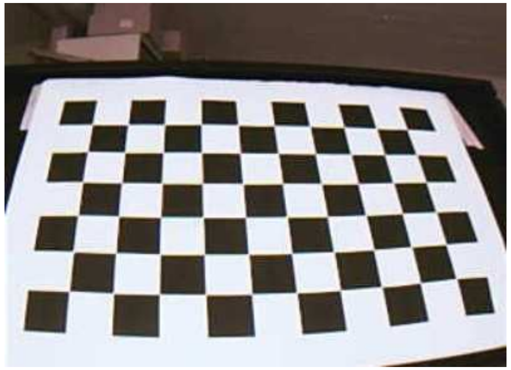
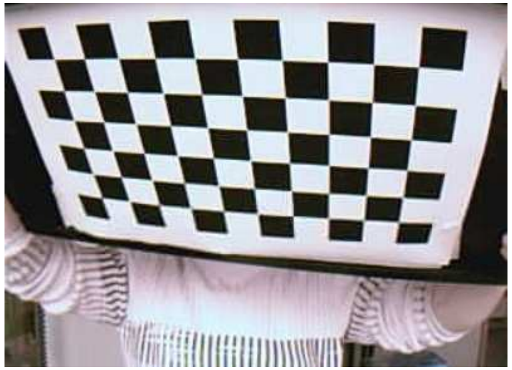
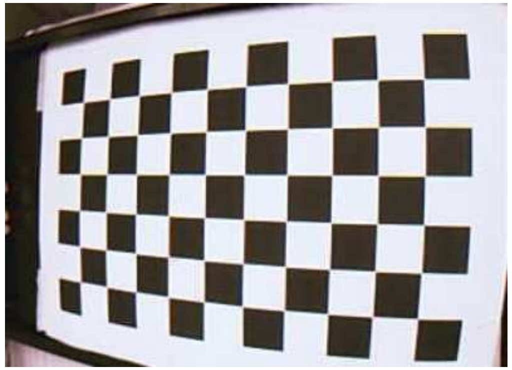
- Lens distortion can be calculated by minimizing a non-linear function

$$\min_{(\mathbf{K},\kappa,\mathbf{R}_i,\mathbf{t}_i)} \sum_i \sum_j \|\mathbf{x}_{ij} - \widehat{x}(\mathbf{K}, \kappa, \mathbf{R}_i, \mathbf{t}_i; \mathbf{X}_{ij})\|^2$$

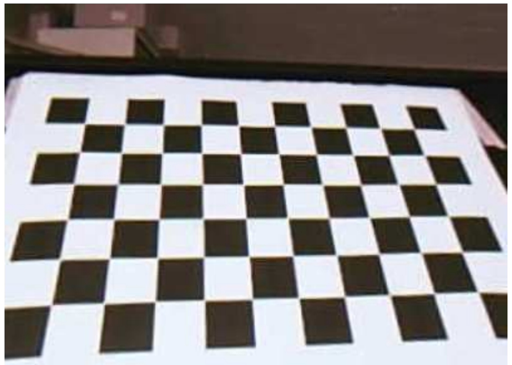
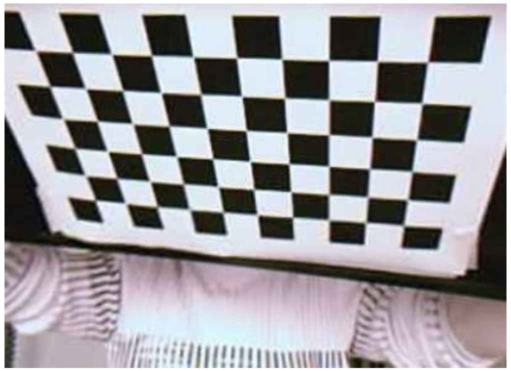
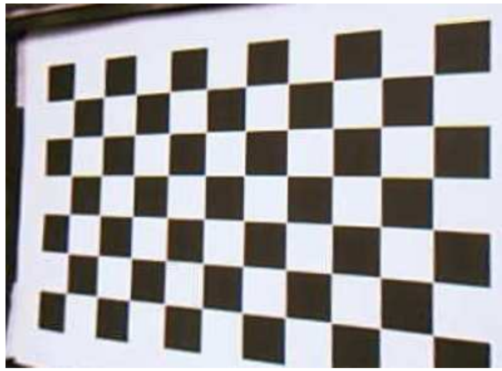
- Estimation of κ using non-linear optimization techniques (e.g. Levenberg-Marquardt)
- The parameters obtained by the linear function are used as starting values

Results: Webcam

- Before calibration:

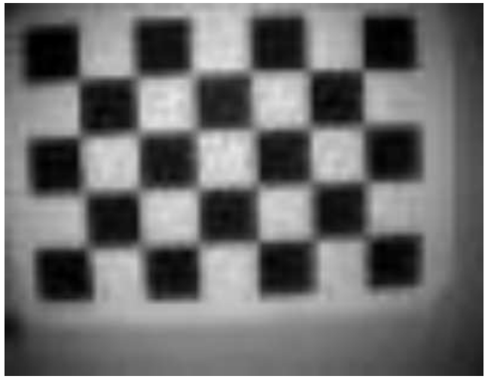
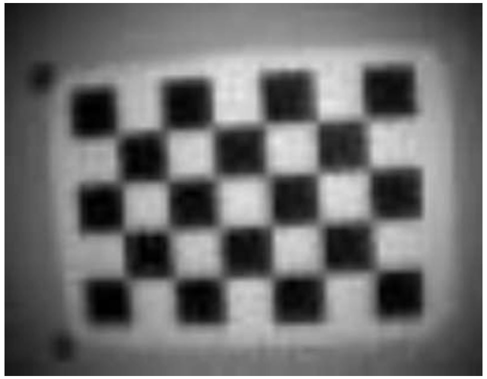
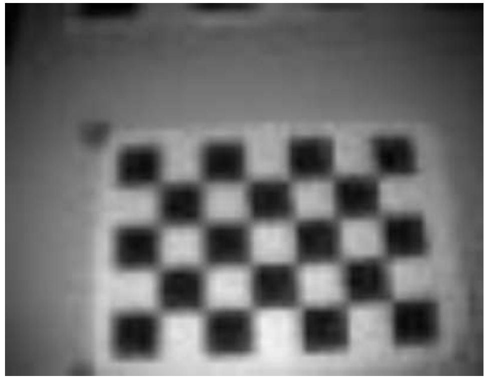




- After calibration:

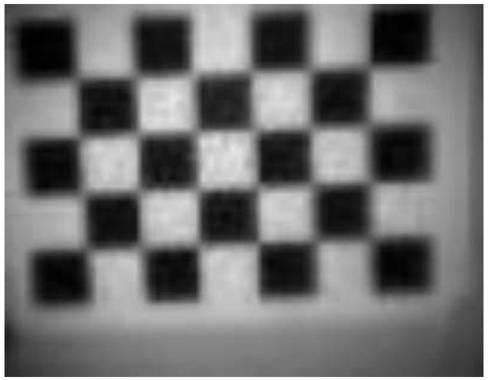
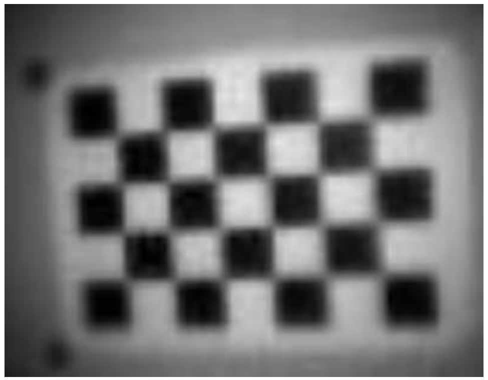
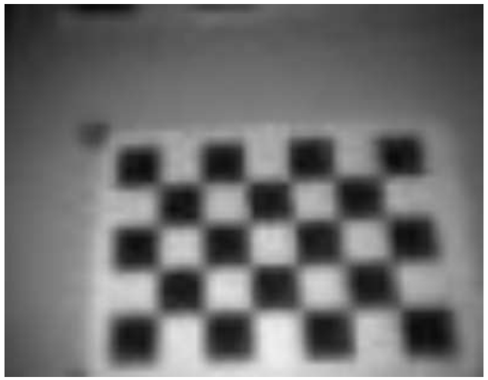




Results: ToF-Camera

- Before calibration:

- After calibration:

Summary

- Pinhole Camera Model
- Non-linear model for lens distortion
- Approach to 2D Calibration that
  - accurately determines the model parameters and
  - is easy to realize