Eye Tracking Methods and Applications @ Clemson
With Slight Emphasis on Interaction

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Abstract

A brief exposé of eye tracking research, education, and service at Clemson University is presented. Spanning about a decade, past work involves visual inspection, eye movement analysis, training, and gaze-contingent displays. Current projects include interaction and offline analysis (e.g., scanpath comparison as a usability metric). Real-time approaches will be discussed with applications to accessibility, videoconferencing, eye movement prediction, and peripheral color degradation.
What is eye tracking?

- Method for calculating what user is looking at.
- Accomplished through computer vision techniques imaging pupil and corneal reflection of (near) infra-red light.
- Performed in real-time, hence applicable to gaze-controlled user interface control (e.g., eyes used as pointer in traditional WIMP metaphor).
- Off-line analysis of real-time recorded eye movements can lead to novel insights into human perception, behaviour, etc.
- Recent technological trends (digital cameras) have once again revitalized the field (permanently this time, it seems).
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Eye Tracking Applications

Research context:

- Eye Tracking Systems
  - Interactive
  - Diagnostic
    - Selective
    - Gaze-Contingent
    - Process Measures
    - Retrospective Think-Aloud
      - Screen-Based
      - Hybrid
      - Model-Based

On-line vs. off-line
- On-line apps ~ interactive, real-time
- Off-line apps ~ diagnostic

Both forms rely on eye movement analysis
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Clemson’s Eye Tracking Laboratory.

- Older equipment as well as modern 4\textsuperscript{th} generation eye trackers (installed Fall ’05).
- Installation supports eye tracking class and experiments.
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Tobii Eye Tracking Workstation

PC Monitor

Tobii ET-1750

Monitor cable

Serial (USB) and Firewire cables

Monitor cable

TCP/IP

Eye-tracking computer

Application computer

Dual-head FP

Monitor cable
Older technology is, by today’s standards, truly cumbersome.

Current eye tracking stations consist of:

1. Tobii ET-1750 eye tracker (50 Hz sampling rate, 1280×1024 17” display, 0.5° accuracy),
2. Sun W2100z server running Windows XP (dual 2.0 GHz AMD Opteron 246 CPUs, 2GB RAM),

All machines are connected to 1Gb/s Ethernet subnet.

Tobii’s SDK and TCP/IP client/server model makes Linux development much simpler than before.

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Determine gaze point \((x_g, y_g, z_g)\)

Given instantaneous eye tracked coordinates, \((x_l, y_l)\) and \((x_r, y_r)\), and head-tracked head position coordinates, \((x_h, y_h, z_h)\), the coordinates of the gaze point, \((x_g, y_g, z_g)\), are:

\[
\begin{align*}
x_g &= (1 - s)x_h + s(x_l + x_r)/2 \\
y_g &= (1 - s)y_h + s(y_l + y_r)/2 \\
z_g &= (1 - s)z_h + sf,
\end{align*}
\]

where \(s = b/(x_l - x_r + b)\), \(b\) is the baseline distance between the left and right eye centers, and \(f\) is the distance to the near viewing plane along the head-centric \(z\)-axis.
Incorporate head position

- Given raw gaze intersection points in 3D, eye movement analysis requires that:
  1. the head position, $h$, is recorded to calculate visual angle,
  2. with two successive intersection points in three-space, $p_i = (x_i, y_i, z_i)$ and $p_{i+1} = (x_{i+1}, y_{i+1}, z_{i+1})$, and the head position at each instance, $h_i$ and $h_{i+1}$, the visual angle is obtained:

$$\theta_i = \cos^{-1} \frac{v_i \cdot v_{i+1}}{\|v_i\| \|v_{i+1}\|}, \quad i \in [0, n),$$

where $n$ is the sample size and $v_i = p_i - \bar{h}$ and $\bar{h}$ is the averaged head position over the sample time period.
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Real-time classification

- An FIR filter, $h_k$, is used for velocity-based saccade detection,

$$\dot{\theta}_i = \frac{1}{\Delta t} \sum_{j=0}^{k} \theta_{i+j} h_j, \quad i \in [0, n - k),$$

expressed in deg/s, where $k$ is the filter length, $\Delta t = k - i$.

- Acceleration is obtained via a subsequent convolution of velocity, $\dot{\theta}_i$, with an acceleration filter, $g_j$,

$$\ddot{\theta}_i = \frac{1}{\Delta t} \sum_{j=0}^{k} \dot{\theta}_{i+j} g_j, \quad i \in [0, n - k),$$

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Acceleration-based filtering

- If $|\ddot{\theta}_i| > A$, the corresponding gaze intersection point $p_i$ is treated as the beginning of a saccade.
- Three additional conditions are evaluated to characterize a saccade, as illustrated below.

1. $|\ddot{\theta}_i| > A$
2. $|\ddot{\theta}_{i+j}| > B$
3. $\text{sgn}(\ddot{\theta}_{i+j}) \neq \text{sgn}(\ddot{\theta}_i)$
4. $T_{min} < j - i < T_{max}$
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![Diagram showing acceleration-based filtering conditions](image-url)
Results: physical cargo bay (upper left), raw data (upper right), 2-tap (lower left), 5-tap (lower right) velocity filter.
Training application

Following analysis, eye movements abstracted further to display expert’s search strategy.

Eye movements constitute *process measures* and augment traditional *performance measures* (speed and accuracy).
Stimulus

Sadasivan’s CHI 05 video.
Outcomes

(a) True positives.
(b) Time to complete.
(c) Fixation points.
(d) Fixation groups.
(e) Fixation duration.

Performance and process measures: relative difference (%).

- Speed-accuracy tradeoff was observed: performance improved while completion time increased.
- Eye movement analysis suggests that training made novice inspectors slower but more deliberate.
Moving to 3D immersive environment

3D cargo bay.

- Visual inspection in 3D CVE.
- Problem: artificial defect and gaze point registration.
Pubs and pics

(Bob Danforth) (Hunter Murphy) (Nathan Cournia)

GCD examples.

- Showed feasibility of gaze-contingent terrain generation (Smart Graphics 2000 (Danforth et al., 2000)).
- Reported 10-fold frame rate speed up (EuroGraphics 2001 (Murphy & Duchowski, 2001)).
- Described GPU-based video-based gaze-contingent display (TOMCCAP 2007 (A. T. Duchowski & Çöltekin, 2007)).
Gaze-Contingent Displays

Lit Review

Pubs and pics

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Deictic reference in a CVE.

- Demonstrated advantage of visual deictic reference in VR for training (ETRA 2004 (A. T. Duchowski et al., 2004)).
Usability engineering

ForceNet usability study and results.

- Collaboration between the military (Navy), industry (EMA), and academia.
- Used eye movements to evaluate user interfaces.
- Navy expects to apply to design of Command & Control (C2) applications.
Usability engineering

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Eye Tracking Methodology class

- Quality of students’ coursework is steadily improving.
- Scanpaths showed user attention switching to exocentric map during 3D navigation in VR (EGVE 2004 (Vembar et al., 2004)).
- Gaze-contingent fisheye lens was shown effective for Fitts’ Law-type gaze selection task (GI 2005 (Ashmore et al., 2005)).
Idelix PDT lens

Ashmore’s GI 05 video.
ETRA brings together top researchers from around the world, now in its 5th year.

ETRA proceedings covers.
Mainly interactive in nature

- Scanpath comparison, average scanpath
- Accessibility
- Videoconferencing (with avatars)
- Gaze guidance for education
- Peripheral color degradation
- Limbus detection for wearable eye tracker
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Motivation

Expert (left) and novice (right) scanpaths, captured at Clemson’s Eye Tracking Laboratory.

- Consider the scanpaths shown above.
- A single, quantifiable comparison metric is desirable.
- Solution: implementation of scanpath comparison algorithm based on string editing (Privitera & Stark, 2000).
String editing was used by Privitera & Stark to compare human fixations with those predicted by automatic means (e.g., Itti’s saliency model (Itti et al., 1998)).

\[
\begin{align*}
  s_1 &= abcfeffgdc \\
  s_2 &= afbffdcdf & \text{start} & \text{cost 0}
\end{align*}
\]

\[
\begin{align*}
  s_1 &= abcfeffgdc \\
  s_2 &= afefdcdf & \text{after substitution of first } b \text{ by } e & \text{cost 1}
\end{align*}
\]

\[
\begin{align*}
  s_1 &= abcfeffgdc \\
  s_2 &= abcfeffdcdf & \text{after insertion of } bc \text{ after first } a & \text{cost 2}
\end{align*}
\]

\[
\begin{align*}
  s_1 &= abcfeffgdc \\
  s_2 &= abcfeffdc & \text{after deletion of last } df & \text{cost 2}
\end{align*}
\]

\[
\begin{align*}
  s_1 &= abcfeffgdc \\
  s_2 &= abcfeffgdc & \text{after insertion of } g & \text{cost 1}
\end{align*}
\]

Example of string editing (Privitera & Stark, 2000).
String editing framework

- Comparison relies on cost of three character operations: *deletion*, *insertion*, and *substitution*.

- Character transformation costs are based on Levenshtein distance, as illustrated below for strings $s_1 = abcfeffgdc$ and $s_2 = afbffdcdf$.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>f</th>
<th>b</th>
<th>f</th>
<th>f</th>
<th>f</th>
<th>d</th>
<th>c</th>
<th>d</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>1</td>
<td>2</td>
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<td>6</td>
<td>7</td>
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</tbody>
</table>

Example of Levenshtein distance calculation.
String editing framework

- Comparison relies on cost of three character operations: deletion, insertion, and substitution.
- Character transformation costs are based on Levenshtein distance, as illustrated below for strings $s_1 = abcfeffgdc$ and $s_2 = afbffdcdf$.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
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<th>f</th>
<th>f</th>
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Example of Levenshtein distance calculation.
String editing implementation

- Implementation relies on constructing an array:

\[ A[i][j] = \min(A[i - 1][j] + 1, A[i][j - 1] + 1, A[i - 1][j - 1] + c(i, j)) \]

where

\[ c(i, j) = \begin{cases} 
0, & s_1[i - 1] = s_2[j - 1] \\
1, & \text{otherwise}
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- Similarity coefficients are stored in a table, the Y-matrix, with one row and column for each image viewed by a subject (for all subjects).
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<table>
<thead>
<tr>
<th>$S_p$</th>
<th>Subj. 1</th>
<th>Subj. 2</th>
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<td>S2P2</td>
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<table>
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<td>R</td>
<td>I</td>
</tr>
<tr>
<td>S2P2</td>
<td>R</td>
<td></td>
</tr>
</tbody>
</table>


- Since complete $Y$-matrices are too large for display, parsing diagrams are used to condense the data.
Parsing diagrams

Parsing diagrams contain averages of similarity coefficients collected from Y-matrices and consist of four main entries:

1. **Repetitive**: same viewer looking at same scene at different times;
2. **Local**: different viewers looking at the same scene;
3. **Idiosyncratic**: same viewer looking at different scenes; and
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A Random entry is included for significance testing.

Structure of parsing diagrams (Privitera & Stark, 2000).
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<table>
<thead>
<tr>
<th>Same Subj.</th>
<th>Diff. Subj.</th>
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</thead>
<tbody>
<tr>
<td>Repetitive</td>
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<td>Global</td>
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<tr>
<td>$S_p$</td>
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Related Work

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  - evaluation of web page design and television news layout (Josephson and Holmes, 2002; 2006),
  - amalgamation of numerous scanpaths into a single, representative (e.g., “expert”) scanpath (Hembrooke et al., 2006),
  - augmentation of string editing with Needleman-Wunsch weighting (West et al., 2006) (often used in bioinformatics); resultant eyePatterns is (not yet) freely available at: <http://www.juliamae.com/eyepatterns/>

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Implementation

- Automatic fixation clustering is required.
- Privitera and Stark (2000) relied on $k$-means clustering, which requires \textit{a priori} estimation of $k$ (number of clusters).
- Santella and DeCarlo (2004) presented an alternative with \textit{mean shift} fixation clustering.
- Mean shift’s only existing user-adjustable parameter is $\sigma_s$, which can epistemically be set to match the extent of the retinal dimension of the human fovea (about 5° visual angle).
- The mean shift process is composed of two stages:
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- The process continues by repeatedly moving a point $\mathbf{x}_i$ to a new location, $\mathbf{s}(\mathbf{x}_i)$, the weighted mean of nearby points, based on the kernel function $K$:

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where $K$ is typically a multivariate zero-mean Gaussian and covariance $\sigma^2 \mathbf{I}$.

- Applying the above to fixations, the following zero-mean spatiotemporal Gaussian kernel can be used:

$$K([\mathbf{x}_i, t_i]) = \exp\left(\frac{x_i^2 + y_i^2}{\sigma_s^2} + \frac{t_i^2}{\sigma_t^2}\right)$$

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Preliminary Results

Distinct scanpaths with overlapping clusters.

- The left scanpath proceeds in a roughly clockwise order, the middle counter-clockwise.
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- Taking overlap and sequential order into account, the resulting labels are generated as $s_1 = abc$ and $s_2 = acd$, as shown at right, indicating (spatial) cluster overlap at two locations.
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Screenshot of EyeWrite: gestural interface (w/Jake Wobbrock @ UW).

- Gestures with gaze—like Palm Pilot’s graffiti.
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3. Although each character stroke is longer than single keyboard key (4 per letter, on average), eye tracker inaccuracy and gaze jitter work in EyeWrite’s favor in this instance.
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2. Our expectations were that time was required to learn the alphabet.
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I saw an ad for this “Eye 2 Eye” webcam “teleprompter”. The ad said this device lets you see the person you’re talking to and that it’s worth the $100. But it just looks like a simple periscope (and a rather bulky one). Eye contact in video conferencing is of course an old problem. There must be a better way...

Bodelin’s “se2e” device.
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Hypothesis

1. Gaze-driven avatars should “solve” eye contact problem.
2. Just need good avatars.
3. Ideally, each person should have a locally-stored texture of their face that can be mapped onto an MPEG-4 “talking head” 3D facial skeleton object.
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Pilot study

1. Results inconclusive.
2. No apparent difference between fixed gaze, networked gaze, and random gaze.
3. Study needs to be re-run...
Pilot study

Avatar conferencing and hotspots.

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In this storyboard for a gaze-guiding adventure physics lesson, the hunter wants to shoot the monkey. As soon as the monkey sees the flash from the gun, it will fall from the tree. At what angle should the hunter aim the gun?

- Objective is to guide attention to the essential components of the problem.
- Initial angle is obtained via $\tan^{-1}\left(\frac{\text{height}}{\text{distance}}\right)$.
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The proposed research examines the following questions:

1. Does gaze-guidance help manage attentional distribution and improve learning?

2. Does gaze-guidance influence affective responses of students?

3. How do individual differences in cognitive ability, prior knowledge, and spatial visualization ability interact with the effects of cueing?
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1. No significant performance results yet...
2. Low power experiment \((n = 10, \text{ between-subjects})\).
4. Four of five participants indicated they found gaze-guidance helpful for finding or clarifying things in the picture.
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Motivation

Real-time spatiochromatic peripheral degradation.

- Peripheral color degradation has not been widely studied.
- Are there implications for computer graphics or compression?
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- Peripheral color degradation has not been widely studied.
- Are there implications for computer graphics or compression?
Per-fragment mipmap LOD bias selection illustration.

GPU implementation is now trivial.
Experiment

Physical setup. Participant viewing red target (left) with a 5°
gaze-contingent color mask and locating the target in color-degraded
field (middle), GCD (right).

- The independent variable was visual window size, visual angle 1°,
  5°, 20°, and control.
Results

Mean search time (in sec; left) and accuracy (right) vs. window size.

- Reduction of peripheral chrominance significantly degrades search performance when window is $\leq 5^\circ$.
- For visual search, vision is more sensitive to peripheral color than spatial detail.
- To achieve performance comparable to that of a uniform color display, a window of $20^\circ$ is required.
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Motivation

Our “DIY” wearable eye tracker from relatively inexpensive (∼$700) off-the-shelf components.
Approach

Eye tracker assembly.

- Basically same hardware design described by Li (2006).
- Inexpensive commercial off-the-shelf (COTS) components.
- Digital video minicams from DejaView (Reich et al., 2004).
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- Erroneous ellipses spanning both pupil and limbus are generated by fitting to all feature points.
- Distinguishing between points on the pupil and limbus reduces the number of erroneous ellipses.

(a) Detection of feature points on both pupil and limbus.
(b) Ellipses fit to sets of 5 points selected at random.
(c) Luminance-based delineation of feature points.
(d) Proper detection of erroneous ellipses.
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Signal modeling

- For networked transmission of eye movement coordinates, first-order velocity-based prediction is required.
- To do so, use the discrete Kalman filter for human interactive motion tracking and adapt a similar model for eye movements.
- The discrete Kalman filter is defined by the state-space model of eye movement motion,

\[ x_{k+1} = \Phi_k x_k + w_k, \]

and the discrete-time process observation (measurement),

\[ z_k = H_k x_k + e_k, \]

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$$z_k = H_k x_k + e_k,$$

where (5 definitions follow):
1. $\mathbf{x}_k$ is the $(n \times 1)$ vector state of the eye movement discrete-time process at time $t_k$ modeling the current gaze position and velocity,

\[
\mathbf{x}_k = \begin{pmatrix}
\mathbf{p}_k \\
\mathbf{v}_k
\end{pmatrix}
\]

2. $\Phi_k$ describes the $(n \times n)$ state transition matrix that relates $\mathbf{x}_k$ to $\mathbf{x}_{k+1}$ in the absence of a forcing function,

3. $\mathbf{w}_k$ is the $(n \times 1)$ timewise uncorrelated zero-mean white noise sequence with known covariance structure,

4. $\mathbf{e}_k$ represents the $(m \times 1)$ measurement error (assumed to be a white sequence with known covariance structure and uncorrelated with the $\mathbf{w}_k$ sequence), and

5. $\mathbf{H}_k$ describes the $(m \times n)$ ideal (noiseless) connection between the state vector $\mathbf{x}_k$ and measurement vector $\mathbf{z}_k$ at time $t_k$,

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Signal filtering

The Kalman recursion is composed of 5 computation steps:

1. Compute gain (blending factor):

   \[ K_k = P_k^{-1} H_k^T (H_k P_k^{-1} H_k^T + R_k)^{-1} \]

   where:
   (a) \( P_k \) is the covariance matrix of the estimation error (the negative superscript denotes projection from the previous step; see below), and
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   \[ \hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \cdot \hat{x}_k^-) \]

   where \( z_k - H_k \cdot \hat{x}_k^- \) is the difference of the real measurement and the measurable components of the process state vector.

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4. Project ahead (prediction step):

\[ \hat{x}_{k+1}^- = \Phi_k \hat{x}_k. \]

5. Obtain best estimate of the error covariance matrix:

\[ P_{k+1}^- = \Phi_k P_k \Phi_k^T + Q_k, \]

where \( Q_k \) is the covariance matrix of the white acceleration,

\[ Q_k = E[w_k w_k^T]. \]

For translational motion of constant velocity and random acceleration, derive:

\[ Q_k = \frac{a^2 \Delta t}{6} \begin{pmatrix} 2I(\Delta t)^2 & 3I\Delta t \\ 3I\Delta t & 6I \end{pmatrix} s, \]

where \( I \) is the \( 2 \times 2 \) identity matrix, \( a \) is the spectral amplitude of the white noise, and \( s \) denotes unit seconds.
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Parameter estimation

- Eye movement analysis (characterization into fixations, saccades, and smooth pursuits), as well as prediction, by the Kalman filter depends on the appropriate specification of the stochastic process, described by the matrices \( P_k, Q_k, \) and \( R_k \).

- The recursive update of the matrix \( P_k \) at time \( t_k \) relies on the \textit{a priori} estimates \( P_k^- \) prior to assimilation of the measurement at time \( t_k \) and the initial determination of \( P^-_0 \).

- The estimation error is defined as \( x_k - \hat{x}^-_k \) where vector \( x_k \) is the (unknown) ideal process state vector. The associated error covariance matrix is then

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P^-_k = E[(x_k - \hat{x}^-_k)(x_k - \hat{x}^-_k)^T].
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- Since \( P_k^- \) is updated during the Kalman recursion, it is sufficient to determine \( P_0^- \), although generally this is a difficult task. For the discrete-time Kalman model of human motion, \( P_0^- \) takes the form

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P_0^- = \begin{pmatrix}
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\sigma_{p,v}^2 & \sigma_{v,v}^2
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where \( \sigma_{p,p} \) is the standard deviation of the position estimation error and \( \sigma_{p,v}^2 = E[(p_k - \hat{p}_k^-)(v_k - \hat{v}_k^-)] \) since \( x_k = (p_k, v_k)^T \).

- Over time, \( E[p_k - \hat{p}_k^-] = E[v_k - \hat{v}_k^-] = 0 \) and position and velocity estimation errors can be assumed to be independent and uncorrelated, leading to \( \sigma_{p,v} = 0 \) (Kohler, 1997).
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Preliminary results

True $x$-position, $\chi^2(dx/dt)$, classification.

- Filter appears to be working fairly well except during saccadic intervals—tends to overshoot prediction.
- Classification appears possible with position-variance used to detect fixations first, then check “velocity-variance” to disambiguate saccades from smooth pursuits.
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For Further Reading II


