Basic elements of 3-D coordinate systems and linear algebra

Coordinate systems are used to assign numeric values to locations with respect to a particular frame of reference commonly referred to as the origin. The number of dimensions in the coordinate system is equal to the number of perpendicular (orthogonal) axes and is also the number values needed to specify a location with respect to the origin.

*One dimension:* the line

*Two dimensions:* the plane

\( (4.5, 2) \)

\( (0, 0) \)
Three dimensions: the universe as we perceive it

(A right handed coordinate system is shown. In a left handed system the direction of the positive z axis is reversed.)
**Points in 3-D space**

The location of a point $P$ in 3-D Euclidean space is given by a triple $(p_x, p_y, p_z)$.

The $x$, $y$, and $z$ coordinates specify the distance you must travel in directions parallel to the the $x$, $y$, and $z$ axes starting from the origin $(0, 0, 0)$ to arrive at the point $(p_x, p_y, p_z)$.

**Vectors in 3-D space**

A vector in 3-D space is sometimes called a directed distance because it represents both

- a direction and
- a magnitude or distance

In this context, the triple $(p_x, p_y, p_z)$ can also be considered to represent

- the direction from the origin $(0, 0, 0)$ to $(p_x, p_y, p_z)$ and
- its length $\sqrt{p_x^2 + p_y^2 + p_z^2}$ is the Euclidean (straight line) distance from the origin to $(p_x, p_y, p_z)$. 


Points and vectors

Two points in 3-D space implicitly determine a vector pointing from one to the other. Given two points P and Q in 3-D Euclidean space, the vector

\[ R = P - Q = (p_x - q_x, p_y - q_y, p_z - q_z) \]

represents the direction from Q to P. Its length, as defined above is the distance, between P and Q. Note that the direction is a signed quantity. The direction from P to Q is the negative of the direction from Q to P. However, the distance from P to Q is always the same as the distance from Q to P.

Example: Let \( V = (8, 6, 5) \) and \( P = (3, 2, 0) \).

Then the vector direction from V to P is : \((3 - 8, 2 - 6, 0 - 5) = (-5, -4, -5)\)
The vector direction from P to V is \((5, 4, 5)\)  
The distance between V and P is: \( \sqrt{25 + 16 + 25} = \sqrt{66} = 8.12. \)
The geometric interpretation of vector arithmetic

Here we work with 2 dimensional vectors to simplify the visual interpretation, but in 3-d the principles are the same.

\[ P = (5, 1) \Rightarrow +5 \text{ in the } x \text{ direction and then } +1 \text{ in the } y \text{ direction} \]
\[ Q = (2, 4) \Rightarrow +2 \text{ in the } x \text{ direction and } +4 \text{ in the } y \text{ direction.} \]
\[ R = P + Q = (7, 5) \]
\[ P = R + (-Q) = R - Q \]
Useful operations on vectors:

We define the sum of two vectors $P$ and $Q$ as the componentwise sums:

$$R = P + Q = (p_x + q_x, p_y + q_y, p_z + q_z)$$

$$(3, 4, 5) + (1, 2, 6) = (4, 6, 11)$$

The difference of two vectors is computed as the componentwise differences:

$$R = P - Q = (p_x - q_x, p_y - q_y, p_z - q_z)$$

$$(3, 4, 5) - (1, 2, 6) = (2, 2, -1)$$

We also define multiplication (or scaling) of a vector by a scalar number $a$

$$S = aP = (ap_x, ap_y, ap_z)$$

$$3 \times (1, 2, 3) = (3, 6, 9)$$

The length of a vector $P$ is a scalar whose value is denoted:

$$\|P\| = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

$$\|(3, 4, 5)\| = \sqrt{9 + 16 + 25} = \sqrt{50}$$

A unit vector is a vector whose length is 1. Therefore an arbitrary vector $P$ may be converted to a unit vector by scaling it by $1 / (\text{its own length})$. Here $U$ is a unit vector in the same direction as $P$.

$$U = \left( \frac{1}{\|P\|} \right) P$$

The inner product or dot product of two vectors $P$ and $Q$ is a scalar number. It is computed by taking the sum of the componentwise products of the two vectors.

$$x = P \cdot Q = (p_x q_x + p_y q_y + p_z q_z)$$

$$(2, 3, 4) \cdot (3, 2, 1) = 6 + 6 + 4 = 16$$

Thus $\|P\| = \sqrt{P \cdot P}$

If $U$ and $V$ are unit vectors and $q$ is the angle between them then:

$$\cos(q) = U \cdot V = V \cdot U$$
Representing vectors in C

There are at least two easy ways to represent a vector:

Array based representation:

We can use the `typedef` facility to create a user defined type called `vec_t`. An instance of `vec_t` is three element double precision array:

```c
typedef double vec_t[3];
```

An instance of `vec_t` is three element double precision array and can be created as shown.

```c
vec_t vec;
```

It is understood that

- `vec[0]` is the x-component (coordinate)
- `vec[1]` is the y-component
- `vec[2]` is the z-component

To make this association explicit we use the `#define` facility

```c
#define X 0
#define Y 1
#define Z 2
```

We can then create an instance of the vector (11, 2, -4) as shown:

```c
vec_t v;

v[X] = 11;
v[Y] = 2;
v[Z] = -4;
```
**Structure based representation**

We can also define a structured type in which the elements are explicitly named

```c
typedef struct vec_type
{
    double x;
    double y;
    double z;
} vec_t;
```

```c
vec_t vec;
```

In this representation, it is explicit that

- `vec.x` is the x-component
- `vec.y` is the y-component
- `vec.z` is the z-component

Religious wars have been fought over which is “correct”. We will refuse to engage in the war, but we will use the array based approach in this course.

Because elements of both the structure and the array are guaranteed to be packed into adjacent memory elements its possible to cheat and use either array or structure notation.

```c
vec_t  v = {1.0, 2.0, 3.0};
double  *w = (double  *)&v;

printf("%lf %lf %lf\n",  v.x, v.y, v.z);
printf("%lf %lf %lf\n",  w[0], w[1], w[2]); 
```

```
1.000000 2.000000 3.000000
1.000000 2.000000 3.000000
```
A library for 3-D vector operations

Since the above operations will be commonly required in the raytracer, you will build a library of functions which we will call `vector.h` to perform them. Here are the function prototypes that must be employed. Because the functions are called many times we will use the `inline` mechanism of `gcc` to improve performance. The `static` qualifier is used to avoid duplicate definition errors at link time when functions are included in `.h` files.

/* Scale a 3d vector */

static inline void vec_scale(
    double fact,      /* Scale factor */
    vec_t  v1,        /* Input vector */
    vec_t  v2);       /* Output vector */

/* Return length of a 3d vector */

static inline double vec_len(
    vec_t  v1);    /* Vector whose length is desired */

/* Compute the difference of two vectors */
/* v3 = v2 - v1 */

static inline void vec_diff(
    vec_t  v1,        /* subtrahend */
    vec_t  v2,        /* minuend */
    vec_t  v3);       /* result */

/* Compute the sum of two vectors */
/* ve = v2 + v1 */

static inline void vec_sum(
    vec_t  v1,        /* addend */
    vec_t  v2,        /* addend */
    vec_t  v3);       /* result */
/* Return the inner product of two input vectors */

static inline double vec_dot(
vec_t v1,       /* Input vector 1 */
vec_t v2);       /* Input vector 2 */

/* Copy one vector to another */

static inline void vec_copy(
vec_t v1,       /* input vector */
vec_t v2);       /* output vector */

/* Construct a unit vector in direction of input */

static inline void vec_unit(
vec_t v1,       /* Input vector */
vec_t v2);       /* output unit vec */

/* Read in values of vector from file */

static inline void vec_read(
FILE  *in,
vec_t  v1);

/* Print values of vector to file */

static inline void vec_print(
FILE  *out,       /* output file */
char  *label,     /* label string */
vec_t  v1);       /* vector to print */
Warning regarding aliased parameters

When parameters are passed using pointers a potentially destructive phenomenon known as aliasing may occur. Here the caller of vec_unit() is requesting that a vector be converted to a unit vector in place.

    vec_unit(v1, v1);

Now suppose the implementation of vec_unit() is as follows:

    static inline void vec_unit(
    vec_t  vin,
    vec_t  vout)
    {
        vout[X] = vin[X] / vec_len(vin);
        vout[Y] = vin[Y] / vec_len(vin);
        vout[Z] = vin[Z] / vec_len(vin);
    }

This looks correct and (assuming vec_len()) is working properly it will work correctly as long as the parameters vin and vout point to different vectors. However, if they point to the same vector incorrect computation will result. If vin and vout point to the same vector the assignment.

    vout[X] = vin[X] / vec_len(vin);

also changes vin[X] Therefore, in the subsequent steps of the computation

    vout[Y] = vin[Y] / vec_len(vin);
    vout[Z] = vin[Z] / vec_len(vin);

vec_len() will generally (but not always) return a different value than in the preceding step. For the computation to work correctly, vec_len() must always return the original length of the input vector.
A correct version of `vec_unit()`

The function can be written correctly (and more efficiently) as.

```c
static inline void vec_unit(
    vec_t  vin,
    vec_t  vout)
{
    double scale = 1.0 / vec_len(vin);
    vec_scale(scale, vin, vout);
}
```

ALL vector functions **must** work correctly with aliased parameters.
A sample test driver for `vector.h`

```c
#include <math.h>
#include <stdio.h>
#include <stdlib.h>
#include <memory.h>
#include "vector.h"

vec_t v1 = {3.0, 4.0, 5.0};
vec_t v2 = {4.0, -1.0, 2.0};

int main()
{
    vec_t v3;
    double v;

    vec_print(stdout, "v1", v1);
    vec_print(stdout, "v2", v2);

    vec_diff(v1, v2, v3);
    vec_print(stdout, "v2 - v1 = ", v3);

    v = vec_dot(v1, v2);
    printf("v1 dot v2 is %8.3lf \n", v);

    v = vec_len(v1);
    printf("Length of v1 is %8.3lf \n", v);

    vec_scale(1 / v, v1, v3);
    vec_print(stdout, "v1 scaled by its 1/ length:", v3);

    vec_unit(v1, v1);
    vec_print(stdout, "unit vector in v1 direction:", v1);

    return(0);
}
```

```
acad/cs102/labs10/lab1 ==> a.out
v1   3.000   4.000   5.000
v2   4.000  -1.000   2.000
v2 - v1 =    1.000  -5.000  -3.000
v1 dot v2 is   18.000
Length of v1 is    7.071
v1 scaled by its 1/ length:   0.424   0.566   0.707
unit vector in v1 direction:   0.424   0.566   0.707
```
Representing $rgb$ data

In the raytracer we will work with three types of $rgb$ data:

- reflective materials
- emissive lights
- pixels

We will use $rgb$ data for all three, but will use different models for the interaction of lights with materials than for the pixmap itself.

As in CPSC 101, for the pixmap data used in the .ppm file, we use an unsigned character representation where $0$ means black and $255$ means maximal brightness.

We will store components of pixel values in arrays of size 3 and use the following #define values to access individual elements:

```
#define R 0
#define G 1
#define B 2
```

```
typedef unsigned char irgb_t[3];
```

For representing lights, reflective materials and their interactions we use:

```
typedef double drgb_t[3];
```

In this representation $0.0$ means black and $1.0$ means maximal brightness. It is possible to produce values $> 1.0$ and as in CPSC 101 these must be clamped to lie within the range $[0, 255.999]$ before converting to $irgb_t$.

The $irgb_t$ will be used only in the final pixmap...
Ray tracing introduction

The objective of a ray tracing program is to render a photo-realistic image of a virtual scene in 3 dimensional space. There are two major components in the process:

The virtual camera

1 - *The viewpoint*  This is the location in 3-d space at which the viewer of the scene is located
2 - *The screen*  This defines a virtual window through which the viewer observes the scene. The window can be viewed as a discrete 2-D pixel array ( pixmap). The raytracing procedure computes the color of each pixel. When all pixels have been computed, the pixmap is written out as a .ppm file

The scene to be viewed

3 - *materials*  One or more material definitions may be associated with each object. The material definition describes how the object interacts with a light. Among other things the material definition defines the color of the object.

4 - *light sources*  Lights themselves are not visible, but they do illuminate objects and may be subject to shadowing. Lights may be white or colored.

5 - *visible objects*  Reflective objects that are illuminated by the light sources
**World and window coordinate systems**

Two coordinate systems will be involved and it will be necessary to map between them:

1. **Window coordinates**
   - the coordinates of individual pixels in the virtual window. These are two dimensional (x, y) integer numbers. For example, if a 400 col x 300 row image is being created, the window x coordinates range from 0 to 399 and the window y coordinates range from 0 to 299. In the raytracing algorithm, a ray will be *fired* through each pixel in the window. The color of the pixel will be determined by the color of the object(s) the ray hits.

2. **World coordinates**
   - the “natural” coordinates of the scene measured in feet/meters etc. Since world coordinates describe the entire scene, these coordinates are three dimensional (x, y, z) floating point numbers.

For the sake of simplicity, we will assume that:

- the **screen** lies in the \( z = 0.0 \) plane.
- the **lower left corner** of the **window** has world coordinates \((0.0, 0.0, 0.0)\).
- the **lower left corner** of the **window** has window (pixel) coordinates \((0, 0)\).
- the location of the **viewpoint** has a positive \( z \) coordinate.
- all objects have negative \( z \) coordinates.
- lights may be located in either *positive or negative* \( z \) space.
Translating from pixel to world coordinates

Problem: Suppose the window is 640 pixels wide x 480 pixels high, and that the dimension of the window in world coordinates is 8 feet wide by 6 feet high. Find the world coordinates of the pixel at column 100 row 40.

Possible Solution: Compute the fraction or percentage of the complete x size that must be traversed to reach column 100. This value is 100/640 = 10/64 meaning column 100 is 10/64 of the way across the window. The x world coordinate of this location is therefore 10/64 of the total world distance across the window or \((10/64)*8 = 10/8 = 1.25\). Similarly the world y coordinate is \((40/480)*6 = (1/12)*6 = 0.5\).

A general formula for the procedure is thus:

\[
\text{world}_x = \text{world}_\text{size}_x \times \text{win}_x / (\text{win}_\text{size}_x)
\]

Thus the desired world coordinate is \((1.25, 0.5, 0.0)\). (Recall the screen lies in the \(z = 0\) plane. Therefore the \(z\) world coordinate of every point in the window is 0.0).

WARNING: Pixel dimensions are stored as integers. You must ensure that the divisions shown above are done in floating point.
An alternative “world view”

If the above approach is used, then the pixel with x coordinate 0 clearly maps to world coordinate 0 as it apparently should. But if we are constructing a 640 pixel image, the maximum pixel coordinate is thus 639. And thus the corresponding world coordinate is:

\[ 8 \times \frac{639}{640} = 7.988 \text{ instead of 8.} \]

We can fix that by changing

\[
world_x = \frac{world_size_x \times \text{win}_x}{\text{win}_x \text{ - 1}}
\]

In this way pixel coordinate 0 maps to world coordinate 0 and pixel coordinate 639 maps to world coordinate 8. But then “nice” pixel coordinates such as 40 and 100 now map to really ugly numbers slightly larger than 1.25 and 0.5! Furthermore the image has no “center” pixel that maps to world coordinate (4.0, 3.0, 0.0)!

We can get back our “nice” numbers and our center pixel by using the above strategy but always making the image size 1 more than a “nice size” (e.g. 801 x 601). Since the computer doesn't really care whether a number is ugly or nice, we will use this formulation.

\[
world_x = \frac{world_size_x \times \text{win}_x}{\text{win}_x \text{ - 1}}
\]

\[
world_y = \frac{world_size_y \times \text{win}_y}{\text{win}_y \text{ - 1}}
\]
Computing the direction of a ray

Problem: Suppose the viewpoint is at location (4, 3, 6) in world coordinates. Compute a unit length vector from the viewpoint through the pixel at column 100 row 40.

Solution: We saw above that the world coordinates of the pixel are (approximately): (1.25, 0.5, 0). From page three we know that two points in 3-D space implicitly determine a vector pointing from one to the other. Given two points P and Q in 3-D Euclidean space, the vector

\[ R = P - Q = (p_x - q_x, p_y - q_y, p_z - q_z) \]

represents the direction from Q to P. Therefore the vector from the viewpoint to the point on the window is \((\text{point – viewpoint})\) or:

\[ (1.25, 0.5, 0) - (4, 3, 6) = (1.25 - 4, 0.5 - 3, 0 - 6) = (-2.75, -2.50, -6.00) \]

The length of this vector is 7.06 and so a unit length vector in this direction is:

\[ (-0.39, -0.35, -0.85) \]

If you have computed the direction correctly the z component of the vector will always be negative. A good plan is therefore to include the line:

\[ \text{assert (direction\[Z\] < 0);} \]

The assert facility will abort your program if the condition is FALSE and will print the module and line number where the problem happened.

You might be tempted to also do:

\[ \text{assert (vec_len(direction)) == 1.0);} \]

but because floating point arithmetic is imprecise that would not be a good idea.
The raytracing algorithm

The complete algorithm for the first version of the raytracer is summarized below:

Phase 1: Initialization

load the model description containing camera, material, object, and light definitions
print the camera, material, object and light descriptions to the stderr

Phase 2: The raytracing procedure for building the pixmap

for each pixel in the window
{
    initialize the color of the pixel to (0.0, 0.0, 0.0)
    compute the direction in 3-d space of a ray from the viewpoint through the pixel
    identify the first (closest) object hit by the ray
    make a copy of the ambient color of the material associated with the object
    scale the copy of the ambient color by 1.0 / distance(from_viewpt, to_hitpt)
    add the scaled value to the color of the pixel.
    convert the d_rgb pixel to i_rgb and store it in the pixmap.
}

Phase 3: Writing out the pixmap as a .ppm file

write .ppm header to stdout
write the image to stdout
Example input file and image

camera cam1
{
    pixeldim 640 480
    worlddim 8 6
    viewpoint 4 3 6
}

material green
{
    ambient 0 5 0
}

material yellow
{
    diffuse 4 4 0
    ambient 5 4 0
    specular 1 1 1
}

plane leftwall
{
    material green
    normal 3 0 1
    point 0 0 0
}

plane rightwall
{
    material yellow
    normal -3 0 1
    point 8 0 0
}

material gray
{
    ambient 2 2 2
}

plane floor
{
    material gray
    normal 0 1 0
    point 0 -0.2 0
}

Our lighting model assumes 3 types of reflectivity: ambient, diffuse, and specular. Initially only ambient will be implemented.

Plane definitions must contain the three attributes shown. Materials must be defined before they are referenced. The value of point is the (x, y, z) coordinates of any point on the plane. The value of normal is a vector perpendicular to the plane in the direction of the viewpoint.

Reflectivity (i.e. color) of objects is specified as floating point values. Values must be chosen in an ad-hoc manner.

camera, material, and plane are entity type identifiers (analogous to int, char, float). Only defined type names are legal in this context.

cam1, green, and leftwall are instance names analogous to variable names in C. Any name may be used here.
The output image

The output produced by the input file on the previous page is shown below. Visible image corruption near the green-gray boundary courtesy of JPEG compression.

The color gradient (which is what provides the “three-D” effect) is achieved by dividing the base ambient reflectivity of the object (0 5 0) by the distance from the view point to the location at which the ray hits the object. Pixels near the green – yellow boundary are more distant from the view point than those near the edges of the images.
We can push the point of intersection of the planes even farther into negative z-space by reducing the z component of the normal from 1 to 0.1. When we do this, the floor triangle becomes larger, and the intersection of the two plains becomes indistinct.

plane leftwall
{
    material green
    normal 3 0 0.1
    point 0 0 0
}
plane rightwall
{
    material yellow
    normal -3 0 0.1
    point 8 0 0
}
List management functions

The raytracer needs a generic list manager capable of managing lists of structures representing:

- materials
- lights
- visible objects

The characteristics of the lists used by the raytracer include the following:

1 - Newly created structures are always added to the end of the appropriate list
2 - Individual structures are never deleted from the list
3 - Lists are always processed sequentially from beginning to end
4 - We desire a single generic mechanism that can manage lists of the three different structure types.

```
struct list_type
{
    link_t *first;
    link_t *last;
    link_t *current;
};

struct link_type
{
    link_t *next;
    void *entity;
};

entity on list
```
List data structures

The link_t structure

The `typedef` facility can be used to create an identifier for a user defined type. The following example creates a new type name, `link_t`, which is 100% equivalent to `struct link_type`

```c
typedef struct link_type
{
    struct link_type *next; /* next link in the list */
    void      *entity; /* the entity(object_t, light_t) */
        /* that this link owns */
}  link_t;
```

- There is a single instance of the `link_t` structure for each element in each list.
- **Note** we must use "official" name `struct link_type` when declaring `next` because `link_t` is not known to be a type definition until 3 lines later!
- As shown in the figure on the previous page each link contains two pointers;
  - One is to the next `link_t` in the list.
  - The other points to the actual entity being managed by the list

- The entity pointer is declared to be of type `void *`,
  - A `void *` pointer is a pointer to something of unknown or generic type.
  - In the raytracer, depending on the list being processed, the entity might be an `object_t`, a `material_t` or a `light_t`
  - `void *` pointers can be freely assigned to other pointers
  - `void *` pointers can never be directly used to access the memory to which they point because the size and type of the location is unknown.

Example:

```c
material_t mat;
material_t *mloc = &mat;
void      *vloc = &mat;
```

Now `vloc` and `mloc` both point to the same instance of a `material_t`, but only `mloc` can be use to access the elements of the `material_t` structure.

```c
mloc->ambient[RED] = 2.2;
vloc->ambient[RED] = 1.0  <--- Won't work. Gives compile error
```
The list_t structure

There is a single instance of the list_t structure for each list.

typedef struct list_type
{
    link_t  *first; /* pointer to first link in list */
    link_t  *last; /* pointer to last link in list */
    link_t  *current; /* current link in list */
} list_t;

- There is a single instance of the list_t structure for each list.
- We will use three lists in the ray tracer: one for materials, one for visible objects, and one for lights.
Implementation

We will take an object oriented approach in building our list manager.

An object consists of a collection of:

- related data items and
- functions or methods that can be used external "users" to manipulate the data items
- external "users" are not allowed to read or write the "private" data items associated with the object.

In C++ (or Java) an object is defined via a class definition.

- The link_t structure becomes a link_t class.
- It keeps the same data items (next and entity) but is augmented by methods that are used to manipulate them.
- The list_t structure would be defined in a separate class definition.

In C an object is represented by a structure that contains the related data items and a collection of functions that manipulate them. External users of the "object" should not directly reference the data items.

For now, our list management module (to be constructed in lab) will include the following function:
The list_init() function used to create a new list. In a true O-O language, each class has a constructor method that is automatically invoked when a new instance of the class is created. The list_init() function serves this role here:

Its mission is to:

1 - malloc() a new list_t structure.
2 - set the first, and current and last elements of the structure to NULL.
3 - return a pointer to the list_t to the caller.

```c
list_t *list_init(
void)
{
}
```
The `list_add()` function must add the element pointed to by `new` to the list structure pointed to by `list`. Its mission is to:

1 - `malloc()` a new instance of `link_t`,
2 - add it to the end of the list,
3 - ensure the `next` pointer of the new link is NULL and
4 - ensure the `next` pointer of the `link_t` that used to be at the end of the list points to the new `link_t`
5 - Set the current pointer to the new `link_t`

Two cases must be distinguished:

1 - the list is empty (`list->first == NULL`)
2 - the list is not empty (`list->first != NULL`)

```c
void list_add(
    list_t *list,
    void     *new)
{
}
```

The `list_reset()` function should set the `current` pointer to the `first` pointer. If the list is empty this will cause the `current` pointer to be set to NULL.

```c
int list_reset(
    list_t *list)
{
}
```
The `list_not_end()` function should return 1 if the `current` pointer is not null and return 0 if the `current pointer is NULL`. Thus, `list_not_end()` should return(0) when either the list is empty or when the `current` pointer is advanced beyond the last link in the list.

```c
int list_not_end(
    list_t *list)
{
}
```

The `list_get_entity()` function should return the address of the entity pointed to by the `link` to which the `current pointer` points. The `list_not_end()` function should be called **BEFORE** `list_get_entity()` is invoked to make sure the `current pointer` points to a valid link! The call to `assert()` will abort the program if `list_get_entity()` is invoked in an improper state.

```c
void *list_get_entity(
    list_t *list)
{
    assert(list->current != NULL);
}
```

The `list_next_link()` function should advance the `current` pointer so that it points to the next link in the list. If the `current` pointer is presently pointing at the last link in the list, then this call will and should set the `current pointer` to NULL. The `list_next_link()` function should never be called when the `current pointer` is already NULL. **Proper use of list_not_end()** will ensure that this doesn't occur.

```c
void *list_next_link(
    list_t *list)
{
    assert(list->current != NULL);
}
```
Deleting a list

The `list_del()` function. This function should process the entire list. For each link in the list, it should

1. invoke the `free()` function to free the *item* the link owns and then
2. it should free the *link_t*.

*Care must be taken not to reference a *link_t* after it has been freed.* When all links and items are free the *list* header itself should be freed.

```c
void list_del(
    list_t   *list)
{

}
```
Processing a list

This code segment shows

- how to define an arbitrary structure that might be managed by the list
- how to ask list init to create a new list.
- how to process the list from first to last

```c
typedef struct entity_type
{
    char e_name[16];
    int e_id;
} e_t;

e_t *eloc;
list_t *elist;

elist = list_init();

/* Load the list */
load_my_list(elist);

/* Now traverse the list printing attributes of the elements */
list_reset(elist); // set current to first element

while (list_not_end(elist))
{
    eloc = (e_t *)list_get_entity(elist);
    printf("%s %d \n", eloc->e_name, eloc->e_id);
    list_next_link(elist);
}
```
Ray tracer data structures: the *ray.h* header file

A common technique in building large programs is to consolidate all required header files into a single header file that can be conveniently included by all source modules. The file *ray.h* will contain most of the important data structures of the ray tracer and will also include the header files needed by all of the modules comprising the system.

/* ray.h */

#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <string.h>
#include <memory.h>
#include <assert.h>

#define NAME_LEN 16        /* max length of entity names */
#define OBJ_COOKIE 12345678
#define MAT_COOKIE 32456123
#define LGT_COOKIE 30492344
#define CAM_COOKIE 49495923
#define MAX_BOUNCES 8       /* Maximum # of ray reflections */
#define AA_SAMPLES 1        /* Used for antialiasing */

/* Local include files containing vector, pixel, and list */
/* definitions and functions */

#include "vector.h"
#include "pixel.h"
#include "list.h"
/* The camera object */

typedef struct camera_type
{
    int    cookie;
    char   name[NAME_LEN];
    int    pixel_dim[2];    /* Projection screen size in pix */
    double world_dim[2];    /* Screen size in world coords */
    vec_t  view_point;      /* Viewpt Loc in world coords */
    irgb_t *pixmap;         /* Build image here */
} camera_t;

typedef struct model_type
{
    camera_t  *cam;
    list_t   *mats;
    list_t   *objs;
    list_t   *lghts;
} model_t;

/* The generic visible object */

typedef struct object_type
{

} object_t;

typedef struct plane_type
{

} plane_t;

/* Function prototypes --- must come last because they */
/* depend on object_t camera_t, etc. */
#include "rayhdrs.h"
An alternative approach

/* ray.h */

#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <string.h>
#include <memory.h>
#include <assert.h>

#define NAME_LEN   16      /* max len of entity/attr names */
#define OBJ_COOKIE 12345678  /* quasi-random cookie values */
#define MAT_COOKIE 32456123  /* used to verify that struct */
#define LGT_COOKIE 30492344  /* pointers are what they */
#define CAM_COOKIE 49495923  /* pretend to be! */

#include "vector.h"   /* vec_t and vector functions */
#include "pixel.h"
#include "list.h"
#include "camera.h"
#include "model.h"
#include "object.h"
#include "plane.h"
#include "sphere.h"

/* Still have to come last !!! */

#include "camhdrs.h"   /* prototypes for intermodule calls */
#include "listhdrs.h"   /* prototypes for intermodule calls */
#include "objhdrs.h"    /* prototypes for intermodule calls */

Neither way is “right” or “wrong”. Factors that influence the choice would be the size of the team working on the program and the personal preference of the programmer.

I use the approach shown on the previous page because it makes it easy to look at all of my data structures all at one time, but you are free to do it any way you wish.

The one approach I DO NOT recommend is including a giant list of header files in every source module. That just makes for unnecessary work when you need to change the list.
Data structures - the big picture

The data structures shown below will all be defined in ray.h

**WARNING:** Some elements of the definitions have been abbreviated and assume the use of the `typedef` construct. See the examples on other pages for these details.

```c
struct model_type
{
    camera_t *cam;
    list_t *objects
    list_t *lights;
    list_t *materials;
};

struct camera_type
{
    int pixel_dim[2];
    double world_dim[2];
    vec_t view_point;
};

struct list_type
{
    link_t *first;
    link_t *last;
    link_t *current;
};

struct link_type
{
    link_t *next;
    void *entity;
};

struct object_type
{
    ;
    void *priv;
};

struct plane_type
{
    vec_t normal;
    vec_t point;
    void *priv;
};

struct sphere_t
{
    vec_t center;
    double radius;
    void *priv;
};
```
The `model_t` data structure

This structure is a container used to reduce the number of parameters that must be passed through the raytracing system. A function having a pointer to the `model_t` structure can access all of the components of the virtual system.

```c
typedef struct model_type
{
    camera_t *cam;    // The camera structure
    list_t   *mats;   // The head of the material list
    list_t   *objs;   // The head of the visible obj list
    list_t   *lgts;   // The head of the light list
}  model_t;
```

As described previously, in code we may use either

```c
struct model_type *model;
```
or

```c
model_t *model;
```

interchangably.
The camera object

A structure of the following type can be used to hold the view point and coordinate mapping data that defines the projection onto the virtual window:

```c
#define NAME_LEN   16
#define CAM_COOKIE 23987237

typedef struct camera_type
{    int    cookie;
    char   name[NAME_LEN];
    int    pixel_dim[2];    /* Projection screen size in pix */
    double world_dim[2];    /* Screen size in world coords */
    vec_t  view_point;      /* Viewpt Loc in world coords */
    irgb_t *pixmap;         /* Build image here */
}  camera_t;
```

The `CAM_COOKIE` value is a completely arbitrary quasi-random identifier that can be used in conjunction with the `assert()` mechanism to detect:

- defective `camera_t` * pointers
- `camera_t` structures that have been corrupted via other pointer errors.

When a camera structure is created the cookie should be initialized

- `cam->cookie = CAM_COOKIE`  

When a function is passed an alleged pointer to a `camera_t` it should be verified

- `assert(cam->cookie == CAM_COOKIE);`

If the value of the expression passed to `assert()` is `false() the program is aborted` and a message issued which provides the module name and line number at which the error was detected.
Camera functions (methods)

camera_init() -

The camera_init() function is responsible for

1. Allocating a camera_t structure with malloc() and initializing the cookie element.
2. Reading in the camera definition data into the camera_t structure and verifying that all attributes have been read.
3. Allocating the pixmap structure that will hold the irgb_t pixels that comprise the image.
4. Saving the address of the camera_t structure in the model_t structure;

```c
void camera_init(
    FILE *in,
    model_t *model,
    int  attrmax)
{
    int attribcount;

    /* Allocate camera structure and store cookie code */

    /* Read attributes into camera data structure */

    attribcount = camera_load_attributes(in, cam);
    assert(attribcount == 3);

    /* Allocate a pixmap to hold the ppm image data */

    /* Save camera pointer in model structure */

}
```
camera_load_attributes -

The `camera_load_attributes()` function is responsible for reading in the values of camera attributes from the input file `in`. It must consume the items shown in red from the camera definition. The word `camera` will have already been read from the file. It must return the number of attributes (in addition to the camera name) that it read.

camera cam1
{
    pixeldim 640 480
    worlddim 8 6
    viewpoint 4 3 6
}

```c
int camera_load_attributes(
    FILE *in,
    camera_t *cam)
{
    char attrib_name[16];
    int count = 0;   // number of items read
    int attrcount = 0;  // total number of attributes

    /* First read camera name string into cam->name */
    count = fscanf(in, "%s", cam->name);
    assert(count == 1);

    /* Now consume "{" */
    count = fscanf(in, "%s", attrib_name);
    assert(count == 1);
    assert(attrib_name[0] == '{');
```
Now consume the attributes and their values.

/* Read first attribute name... */

    count = fscanf(in, "%s", attrib_name);
    assert(count == 1);

/* '}' means end of the camera definition */

    while (attrib_name[0] != '}')
    {
        if (strcmp(attrib_name, "pixeldim") == 0)
        {
            count = fscanf(in, "%d %d",
                            &cam->pixel_dim[X], &cam->pixel_dim[Y]);
            assert(count == 2);
            attrcount += 1;
        }
        else
        {
            fprintf(stderr, "Bad camera attribute: %s \n", attrib_name);
            exit(1);
        }

        /* Read next attribute name */

        fscanf(in, "%s", attrib_name);
    }

    return(attrcount);
}
The `camera_print()` function is responsible for printing the camera attributes.

```c
void camera_print(
   camera_t *cam,
   FILE   *out)
{
   assert(cam-&gt;cookie == CAM_COOKIE);
}
```

Given input that looks like:

```plaintext
camera cam1
{
   pixeldim  800 600
   worlddim   8 6
   viewpoint 4 3 3
}
```

Your output should look like:

```plaintext
camera         cam1
pixeldim       800   600
worlddim       8.0   6.0
viewpoint      4.0   3.0   3.0
```
The `camera_getdir()` function is responsible for computing a unit length vector pointing from the viewpoint to the (x, y) pixel coordinates passed in as parameters. This is done in three steps:

1. Convert pixel coordinates to world screen coordinates
2. Compute vector from viewpoint to world screen coordinate
3. Convert vector to unit length vector.

```c
void camera_getdir(
    camera_t *cam,
    int      x,           /* pixel coordinates */
    int      y,
    vec_t    uvec)        /* Unit vector to be filled in */
{
    assert(cam->cookie == CAM_COOKIE);
}
```
camera_store_pixel() -

The camera_store_pixel() function is responsible for converting a pixel from drgb_t to irgb_t and storing it in the pixmap associated with the camera.

This is done in four steps:

1. multiply the elements of the drgb_t pixel by 255.0 and add 0.5 for rounding
2. if any element is < 0.0, set it to 0.0. if any element is > 255 set it to 255. This computation must all be done in floating point.
3. compute the address of the irgb_t pixel. Remember that .ppm images have upper left origin, but the ray tracer has a lower left origin. Therefore, the (row, col) position in the pixmap corresponding to pixel coordinates (y, x) is (cam->pixel_dim[1] - y - 1, x). See the camtest.c program for an example of how to convert (x, y) coordinates to an irgb_t* pointer that can be used to access the pixmap.
4. store the scaled and clamped values of the drgb_t pixel in the pixmap.

```c
void camera_store_pixel(
    camera_t   *cam,
    int      x,
    int      y,
    drgb_t   pix)
{
    assert(cam->cookie == CAM_COOKIE);
}
```

camera_write_image()

This function should use fprintf() to write the .ppm header and then use a single call to fwrite to write the entire pixmap.

```c
void camera_write_image(
    camera_t *cam,
    FILE     *out)
{
}
```
Testing the camera module

In building a large program such as a ray tracing system it would be the height of insanity to write the complete program, link it together and then test it. It would have thousands of errors and their interactions would be so complex that it would take forever to find them all.

Hence it is common and necessary practice to write modules whose whole mission in life is to perform standalone testing of components.

```c
#include "ray.h"

int main()
{
    model_t   model;
    model_t   *mod = &model;
    camera_t  *cam;
    char      entity[16];
    vec_t     uvec;
    irgb_t    *ipix;
    drgb_t    dpix;
    int       row;
    int       col;
    int       x = 20;
    int       y = 50;

    /* Consume word camera from the model description */
    fscanf(stdin, "%s", entity);

    /* Create new camera object */
    camera_init(stdin, &model, 0);
    cam = mod->cam;
    assert(cam->cookie == CAM_COOKIE);

    /* Make entire pixmap a dark gray color */
    memset(cam->pixmap, 0x40, sizeof(irgb_t) *
            cam->pixel_dim[0] * cam->pixel_dim[1]);

    /* Print camera attributes */
    camera_print(cam, stderr);
}
```
/* Verify getdir works */

camera_getdir(cam, 0, 0, uvec);
vec_print(stderr, "unit_vector: ", uvec);

camera_getdir(cam, 300, 450, uvec);
vec_print(stderr, "unit_vector: ", uvec);

/* Store drgb_t equivalent to (64, 0, 0) */
dpix[R] =  0.25;
dpix[G] = -0.3;
dpix[B] =  1.2;

camera_store_pixel(cam, x, y, dpix);

/* Retrieve it directly and verify its correct --- Note that this */
/* code can be used as a model for writing camera_store_pixel */
row = cam->pixel_dim[1] - y - 1;
col = x;

ipix = cam->pixmap + row * cam->pixel_dim[0] + col;

fprintf(stderr, "ipix is %d %d %d \n",
        (*ipix)[R],
        (*ipix)[G],
        (*ipix)[B]);

/* Put visible bright green dot in the middle of the picture */
dpix[R] =  0.0;
dpix[G] =  1.3;
dpix[B] =  0.0;

camera_store_pixel(cam, cam->pixel_dim[0] / 2,
                    cam->pixel_dim[1] / 2,  dpix);

/* Write out the image */

camera_write_image(cam, stdout);

return(0);
}
The material object

Each visible object must be associated with at least one `material_type` which defines the way the surface of the object interacts with light in the scene. At the simplest level, the material definition can be thought of as specifying the color of the object in `drgb_t (r, g, b)` units.

```c
typedef struct material_type
{
    int     cookie;          /* material_t cookie */
    char    name[NAME_LEN];  /* light_blue for example */
    drgb_t  ambient;         /* Reflectivity for materials */
    drgb_t  diffuse;
    drgb_t  specular;
}  material_t;
```

There are three components to the light interaction model:

- **ambient** – specifies how the object reflects light that is present in the scene but is *not* emanating from any particular light source. This is how we will initially illuminate our scenes. The visible color of a pixel will be the ambient reflectivity divided by the distance from the viewpoint to the location in 3D space where the ray hits the object.

- **diffuse** – specifies how the object reflects light that *does* emanate from specific light sources. This will have no effect until we implement light sources. As with ambient lighting, diffuse lighting simulates the physical process by which a photon is absorbed by the material and a new photon having energy (color) dependent upon the atomic structure of the material is emitted.

- **specular** – specifies the degree to which the object acts like a mirror (incoming light is precisely reflected (instead of being diffused) with the angle of incidence being equal to the angle of reflection).

It is possible to create models that are physically unrealizable. We can define an object that reflects ambient light as red and diffuse light as green! But no physical object exists that operates in such a way.
Material functions (methods)

*material_init()* -

The *material_init()* function is responsible for

1) allocating a *material_t* structure with *malloc()* and initializing the *cookie* element. Unlike the camera, *material attributes are all optional*. So the *memset()* function should be used to initialize the entire structure to 0 before initializing the *cookie*.
2) reading in the material attributes into the *material_t* structure.
3) adding the address of the *material_t* structure to the *mats* list of the *model_t* structure;

```c
/**
 * Create a new material description */

void material_init(
    FILE       *in,
    model_t    *model,
    int        attrmax)    // ignore
{
    material_t *mat;

    /* malloc() a material_t structure, use memset() to */
    /* initialize it to 0 and store the MAT_COOKIE */

    /* Load attributes as in camera.c */
    /* Unlike the camera the number of attributes is */
    /* optional. Attributes should be initialized to 0.0 */

    material_load_attributes(in, mat);

    /* Ask list_add to add the material entity to the end */
    /* of the mats list in the model structure. */
}
```

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material_getbyname() -

This function must search the list of materials looking for a material for which `mat->name` matches the color specified in the name parameter:

/**/
/* Try to locate a material by name */

material_t *material_getbyname(
    model_t *model,
    char    *name)    // requested material name (e.g. yellow)
{
    material_t *mat;

    for each mat in the model->mats list
    {
        assert(mat->cookie == MAT_COOKIE);
        if (mat->name matches name) // use strcmp here
            return(mat)
    }
    return(NULL);
}
material_list_print() -

The \texttt{material_list_print()} function processes the entire material list. It should call \texttt{material_item_print()} to print each item.

\begin{verbatim}
/**
/* Produce a formatted dump of the material list */

void material_list_print(
    model_t *model,
    FILE *out)
{
    for each mat in the model->mats list
    {
        assert(mat->cookie == MAT_COOKIE);
        material_print(mat, out);
    }
}
\end{verbatim}

material_print()

The \texttt{material_print()} function should print a formatted version version of the material structure. The format should be consistent with that produced by \texttt{camera_print()}.

\begin{verbatim}
static inline void material_print(
    material_t *mat,
    FILE *out)
{
}
\end{verbatim}
The `material_getters()`

These functions simply copy reflectivity of the `material_t` structure that is passed in to the `dest drgb_t` parameter. You can accomplish this with a single call to the `pix_copy()` function (found in `pixel.h`).

Since the caller of this function must already have a pointer to the `material_t` structure, it may seem like useless overhead to call `material_getambient()` instead of just referencing `mat->ambient[R]` directly.

Nevertheless, there are two good reasons for doing this:

1. In a true O-O language an external user of the `material` class will not be permitted to directly access its private data attributes. That is, trying to directly reference `mat->ambient[R]` will cause a compile time error!

2. These functions also provide the basis for what is called polymorphic behavior. Polymorphism allows us to provide a "default" behavior that can be replaced with a specialized behavior as required. These are the functions that provide the default behavior.

```c
void material_getambient(
    material_t *mat,
    drgb_t      dest)   /* fill in ambient reflectivity here */
{
}

void material_getdiffuse(
    material_t *mat,
    drgb_t      dest)   /* diffuse here */
{
}

void material_getspecular(
    material_t *mat,
    drgb_t      dest)   /* specular here */
{
}
```
Testing the material module

As with the camera module, we will build a test driver to test our material functions in isolation.

/* mattest.c */
#include "ray.h"

int main()
{
    model_t    mod;
    model_t    *model = &mod;
    material_t *mat;
    char       entity[16];
    drgb_t     dpix;
    int        count;

    /* Create a material list */
    model->mats = list_init();

    /* Input should consist only of material definitions */
    count = fscanf(stdin, "%s", entity);

    /* but there can be any number of material defs in the file */
    while (count == 1)
    {
        /* create material_t structure and read attributes */
        material_init(stdin, model, 0);

        /* this test is designed to ensure that list_add pointed current to the material just loaded */
        mat = (material_t *)list_get_entity(model->mats);
        assert(mat->cookie == MAT_COOKIE);
        fprintf(stderr, "loaded %s \n", mat->name);
        count = fscanf(stdin, "%s", entity);
    }
}
/* Have read them all in .. now try to print them */

material_list_print(model, stderr);

/* See if we can find the first in the list */

mat = material_getbyname(model, "blue");
assert(mat->cookie == MAT_COOKIE);
fprintf(stderr, "found %s \n", mat->name);

material_getamb(mat, dpix);
vec_print(stderr, "ambient is: ", dpix);

/* See if we can find the last one */

mat = material_getbyname(model, "yellow");
assert(mat->cookie == MAT_COOKIE);
fprintf(stderr, "found %s \n", mat->name);

material_getamb(mat, dpix);
vec_print(stderr, "ambient is: ", dpix);

/* See what happens if we try to find a non-existent element */

mat = material_getbyname(model, "chartreuse");
assert(mat == NULL);

return(0);
}
The generic object structure

Even though C is technically not an Object Oriented language it is possible to employ mechanisms that emulate both the inheritance and polymorphism found in true Object Oriented languages.

*Inheritance* -

The `object_t` structure serves as the generic “base class” from which specializations such as `planes` or `spheres` are derived. As such, it carries only the attributes that are common to the all derived objects.

Specializations *inherit* the attributes of the classes from which derived. Specialized attributes of a `plane` are carried by a `plane_t` structure. Specialized attributes of a `sphere` are carried by a `sphere_t` structure. The `priv` pointer of the `object_t` provides a link to the `plane_t` or `sphere_t` and is thus declared as `void *`.

*Polymorphism* -

*Polymorphic behavior*, in which the base `object_t` class provides a default behavior that can be overridden by specializations of the object, is achieved by the use of *function pointers* embedded in the `object_t` (or its subordinate specialization.) These can be initialized to point to functions that provide a *default* behavior but may be overridden as needed when an esoteric object such as a `tiled plane` must substitute its own “method”.

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Example of inheritance

An inheritance hierarchy is based upon the principle of increased specialization. In its "purest" form, inheritance can be represented by a proper tree as shown below.

- The base class carries attributes that are common to all classes and
- virtual functions that may or may not be overridden.
- Attributes that are specific to a particular entity plane/normal or sphere/radius are not defined in the base class
- The derived class inherits the attributes of classes above it in the class hierarchy
- The specialization can continue over multiple levels
- The amount of "new stuff" required in the implementation of the derived class can range from trivial (ellipsoid_t, pfplane_t) to moderate (plane_t, sphere_t) to fairly complex (texplane_t)
The generic object structure

typedef struct object_type
{
    int     cookie;
    char    obj_type[NAME_LEN]; /* entity type plane, sphere,.. */
    char    obj_name[NAME_LEN]; /* entity instance name, floor */

    /* Function pointers that can be overridden to provide polymorphic */
    /* behavior.. */

    void    (*printer)(struct object_type *, FILE *);
    double  (*hits)(struct object_type *, vec_t, vec_t);
    void    (*ambient) (struct material_type *, drgb_t );
    void    (*diffuse) (struct material_type *, drgb_t );
    void    (*specular)(struct material_type *, drgb_t );

    /* Pointer to associated material structure */

    material_t *mat;

    /* Data associated with last hit point */

    vec_t   last_hit;       /* Last hit point */
    vec_t   last_normal;    /* Normal at last hit point */

    void    *priv;          /* Private type-dependent data */
} object_t;
Declaration of derived object types

The specific characteristics of derived object types must be carried by structures that are specific to the object type being described. The priv pointer of the base class object_t is used to connect the generic instance to the esoteric instance. This connection is automatic and invisible in a true OO language but is manual and visible in C.

Notice that the process of refinement or specialization can continue over multiple levels. The priv pointer of the plane structure may point to an fplane (bounded rectangular plane) refinement. Or a tplane (tiled plane) refinement.

```c
/* This structure carries the attributes */
/* of an infinite (unbounded) plane */

typedef struct plane_type
{
    vec_t normal;  /* vector perpendicular to plane */
    vec_t point;   /* any point on the plane */
    double ndotq;  /* dot product of normal and point */
    void *priv;    /* Data for specialized types */
} plane_t;

/* Sphere */

typedef struct sphere_type
{
    vec_t center;                  
    double radius;                
    vec_t scale;                  /* for ellipsoids */
    void *priv;                   
} sphere_t;
```
Pointers to functions

Pointer variables may hold the address of a function and be used to invoke the function indirectly:

```c
#include <stdio.h>

int adder(
    int a,
    int b)
{
    return(a + b);
}

int main()
{
    int (*ptrf)(int, int); // declare pointer to function
    int sum;

    ptrf = adder;          // point it to adder (note no &)
                          // is needed (but it doesn't hurt))

    sum = (*ptrf)(3, 4);    // invoke it (*ptrf) parens req'd!
    printf("sum = %d \n", sum);
    return(0);
}

==> a.out
sum = 7
Function pointers as do-it-yourself polymorphism

Recall the the `object_t` structure contains function pointers:

```c
typedef struct object_type
{
    int     cookie;
    char    obj_type[NAME_LEN]; /* entity type plane, sphere,.. */
    char    obj_name[NAME_LEN]; /* entity instance name, floor */
}
/* Function pointers that can be overridden to provide polymorphic */
/* behavior..                                                      */
    void    (*printer)(struct object_type *, FILE *);
    double  (*hits)(struct object_type *, vec_t, vec_t);
    void    (*ambient) (struct material_type *, drgb_t );
    void    (*diffuse) (struct material_type *, drgb_t );
    void    (*specular)(struct material_type *, drgb_t );

These pointers must be set in `object_init()` to provide the default behavior. The elements on the right side of the equal sign:

- must be the names of functions having
- parameters that match the arguments in the above prototypes

```c
obj->printer = object_print;  // These must be functions
obj->hits    = object_no_hit; // ... with matching parms
obj->ambient = material_get_ambient;
    etc.
```

The `plane_init` function must override these default settings providing its own functions that implement carry the characteristic behavior of the plane. In this way we can emulate polymorphic behavior in the C language.

```c
obj->printer = plane_print;
obj->hits    = plane_hits;
```
Implementing polymorphic functions

The mission of a hits function is to determine if a ray fired from location base in unit direction dir hits object obj.

Needless to say a completely different strategy is required to determine if a ray intersects a plane and if it intersects a sphere.

Therefore each visible object must provide its own hit testing function and override the default function (which always returns miss).

All of the hits functions have the same parameters as the prototype in the struct object_type:

```c
double (*hits)(struct object_type *, vec_t, vec_t);
```

```c
double object_no_hit(
    object_t *obj,      /* Candidate object */
    vec_t *base,        /* Start point of ray */
    vec_t *dir)         /* MUST be unit vector */
{
    return(-1.0);      // negative distance means miss.
}
```

```c
double plane_hits(
    object_t *obj,      /* Candidate object */
    vec_t *base,        /* Start point of ray */
    vec_t *dir)         /* MUST be unit vector */
{
}
```

```c
double sphere_hits(
    object_t *obj,      /* Candidate object */
    vec_t *base,        /* Start point of ray */
    vec_t *dir)         /* MUST be unit vector */
{
}
```
**Invoking a polymorphic function**

When a function pointer is contained in a structure, and an entity holds a pointer to the structure, the polymorphic function is called in the following way.

The actual arguments passed to the function must be the same in number and type as declared in the function pointer and in the actual implementation of the function.

```c
dist = obj->hits(obj, ray_base, ray_dir);
```

Note that the caller of the polymorphic function does not know what actual function is being invoked.
The object.c module

This module contains functions used in initializing and printing the generic object.

object_init() -

This function performs operations analogous to camera_init() and material_init(). An object definition is shown below. The token "sphere" will be consumed prior to object_init being called. The object_init() function is responsible for consuming the data shown in red. The remainder of the attributes will be consumed by sphere_init().

```
sphere            center
material  steelblue
center      4.0  1.0  -6.0
radius          5.0
```

```c
void_t object_init(
    FILE       *in,
    model_t    *model)
{
    object_t   *obj;
    material_t *mat;
    char buf[NAME_LEN];
    int count;

    /* Create a new object structure and zero it */

    obj = malloc(sizeof(object_t));
    assert(obj != NULL);

    memset(obj, 0, sizeof(object_t));
    obj->cookie = OBJ_COOKIE;
```
/* Read the descriptive name of the object */
/* left_wall, center_sphere, etc.          */

count = fscanf(in, "%s", obj->obj_name);
assert(count == 1);

/* Consume the delimiter { */

count = fscanf(in, "%s", buf);
assert(buf[0] == '{');

/* The first attribute must be material */

count = fscanf(in, "%s", buf);
assert(count == 1);
assert(strcmp(buf, "material") == 0);

/* Now get the name of the material (blue, green, etc) */

count = fscanf(in, "%s", buf);
assert(count == 1);

/* If the material is defined, save a pointer to the */
/* mat structure in the object structure. Failure to */
/* find the material is a fatal error. */

mat = material_getbyname(model, buf);
assert(mat != NULL);

obj->mat = mat;
/* Initialize default handlers */

    obj->printer = object_print;
    obj->hits = object_no_hit;
    obj->ambient = material_getamb;
    obj->diffuse = material_getdiff;
    obj->specular = material_getspec;

/* Finally add the object to the list */

    list_add(model->objs, (void *)obj);

}
object_no_hit() -

This function just returns the code that ray missed the object. As such it should always be overridden. It is provided to provide warning and avoid segfaults in case a developer of a new object type fails to establish a hits function.

```c
double object_no_hit(
    object_t *obj,      /* Candidate object */
    vec_t  *base,      /* Start point of ray */
    vec_t  *dir)      /* MUST be unit vector */
{
    fprintf(stderr, "Object %s failed to provide hit func \n", 
           obj->object_name);

    return(-1.0);    // negative distance means miss.
}
```

object_print() -

The object_print() function should print the object_type and object_name along with the word "material" and the name of the material. The format should be consistent with other printers.

```
sphere       center       <---- printed by object_print
material     steelblue
center       4.0   1.0  -6.0 <- printed by sphere print
radius       5.0

double     object_print(
    object_t *obj,      /* Candidate object */
    FILE       *out)   /* MUST be unit vector */
{

    ---------------

}
```
object_list_print() -

The object_list_print() function processes the entire object list. It should call the polymorphic function obj->printer() to print each object.

NOTE: object_list_print should call the polymorphic printer obj->printer and NOT CALL object_print() directly

/**/
/* Produce a formatted dump of the material list */

void object_list_print(
model_t *model,
FILE *out)
{

for each obj in the model->objs list
{
    assert(obj->cookie == OBJ_COOKIE)
    invoke polymorphic printer method
}
}
The plane.c module

The infinite plane is the simplest type of visible object. It is useful in building "floors", "ceilings", and "walls". A plane in 3-D spaces is defined by:

- The location of any point on the plane.
- A vector (called the normal) that is perpendicular to the plane

```c
typedef struct plane_type
{
    vec_t   normal;        /* read from model description */
    vec_t   point;         /* read from model description */
    double  ndotq;         /* normal dot point    */
    void    *priv;         /* Data for specialized types */
}  plane_t;
```
The `plane_init()` function has several responsibilities that are described below.

In a true object oriented language, the `plane_type` will be a specialization of the `object_type` and when a new instance of `plane_type` is created the constructors for BOTH the plane and object classes will be AUTOMATICALLY invoked in top down order. When simulating inheritance with C we must

- explicitly invoke the constructors or each element in the hierarchy and
- link the structures that represent them together.

In C++ this will implicitly require that generic attributes appear first in the model definition and so we will implement our pseudo-constructors in a compatible way.
The `plane_init()` function works in a way analogous to `camera_init()` and `material_init()`. One difference is that it must interact with `object_init()`.

```c
void plane_init(
    FILE *in,
    model_t *model,
    int attrmax)       // maximum number of attributes
{
    plane_t  *pln;
    object_t *obj;
    int       count;

    /* Call the object_init() function to create the object_t */
    /* and process the "material" attribute */

    /* Use list_get_entity() to make obj point to the newly */
    /* created object_t structure. Your list_add() function */
    /* must set current to the last element in the list for */
    /* this to work correctly. */

    /* malloc a plane_t structure and set the priv pointer */
    /* in the object_t structure to point to the plane_t */
```
/* Store the word "plane" in the object_type field of */
/* the object_t structure. Use the strcpy() function */

/* Ask plane_load_attributes to load the attributes */
/* Attributes are normal and point */

count = plane_load_attributes(in, pln);
assert(count == 2);

/* Set obj->hits to plane_hits() function and */
/* obj->printer to plane_print() */

/* pre-compute ndotq */

}
**plane_load_attributes** -

This function works just like your other attribute loaders. It must return the number of attributes loaded,

```c
int plane_load_attributes(
    FILE    *in,
    plane_t *pln)
{
    ----- like camera_load_attributes ----- 
}
```

**plane->print**

Each object specific printer is responsible for first calling its parent's print function.

```c
void plane_print(
    object_t *obj,
    FILE     *out)
{
    plane_t *pln;

    /* Print generic attributes */
    object_print(obj, out);

    /* Recover pln pointer from object_t and print */
    /* point and normal in usual format */
}
```

Output should look like

```
plane        floor           <- printed by object_print()
material     gray
normal       0.0   1.0   0.0 <- printed by plane_print()
point        0.0  -0.1   0.0
```
**Hit functions**

Given the viewpoint, ray direction and a pointer to an `object_t` the mission of a *hit* function is to determine if the ray hits the object. If it does, the *hit point and the normal vector at the hit point* should be stored in the `object_t`, and the distance to the hit point returned to the caller. The function should return -1.0 on a miss.

Given $V$, $D$ and an object structure $O$ the mission of a hit function is to determine if a ray based at $V$ traveling in direction $D$ hits $O$.

*All* points on the ray may be expressed as a function of a single parameter $t$ where $t$ is the *distance* along the ray from the viewpoint. Every point $P$ on the ray may thus be expressed as:

$$V + tD \quad \text{for} \quad -\infty < t < \infty \quad \text{for some} \quad t$$

If $P$ is a point on the ray the following relations hold:

- **Distance to point:** $t = \| P - V \|$
- **Location of point:** $P = V + tD$
General Quadric Surfaces

These surfaces are so named because the variables $x, y, \text{and } z$ take on at most the power of two. The general equation for the quadric is given below:

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz = J$$

We will start with two of the simpler ones:

The sphere:
$$x^2 + y^2 + z^2 = r^2$$

and its relative the ellipsoid:
$$Ax^2 + By^2 + Cz^2 = r^2$$

The plane:
$$Gx + Hy + Iz = J$$

for the plane

- the plane normal $N$ is $(G, H, I)$ and
- $J$ is chosen so that the plane passes through the specified point $Q$.

Quadric surfaces are “nice” in a raytracing environment because the intersection of a ray with the surface may always be found by solving, at worst, a quadratic equation.
Determining if a ray hits a plane

This basic strategy will be used in all hits functions:

0 - Assume that $V$ represents the start of the ray and $D$ is a unit vector in its direction.

1 - Derive an equation for an arbitrary point $P$ on the surface of the object.

2 - Recall that all points on the ray are expressed as $V + tD$.

3 - Substitute $V + tD$ for $P$ in the equation derived in (1).

4 - Attempt to solve the equation for $t$.

5 - If a solution $t_h$ can be found, then $H = V + t_h D$.

A plane in three dimensional space is defined by two parameters:

A normal vector $N = (n_x, n_y, n_z)$

A point $Q = (q_x, q_y, q_z)$ through which the plane passes.

A point $P = (p_x, p_y, p_z)$ is on the plane if and only if:

$$N \cdot (P - Q) = 0$$

because, if the two points $P, Q$ lie in the plane, then the vector from one to the other $(P - Q)$ also lies in the plane and thus it is necessarily perpendicular to the plane's normal.

We can rearrange this expression to get:

$$N \cdot P - N \cdot Q = 0$$

$$N \cdot P = N \cdot Q \quad (1)$$

Note that in this equation $N$ and $Q$ are known attributes of the plane and $P$ is the unknown. Recall that the location of any points on a ray based at $V$ with direction $D$ is given by:

$$V + tD$$

Therefore we may replace the $P$ in equation (1) by $V + tD$ and get:

$$N \cdot (V + tD) = N \cdot Q \quad (2)$$
Some algebraic simplification yields allow us to solve this for $t$

\[ N \cdot (V + tD) = N \cdot Q \quad (2) \]

\[ N \cdot V + N \cdot tD = N \cdot Q \]

\[ N \cdot tD = N \cdot Q - N \cdot V \]

\[ t \cdot (N \cdot D) = (N \cdot Q - N \cdot V) \]

\[ t_h = (N \cdot Q - N \cdot V) / (N \cdot D) \quad (3) \]

The \textit{location of the hitpoint} that should be stored in the \texttt{object_t} is thus:

\[ H = V + t_hD \]

The \textit{normal at the hitpoint} which must also be saved in the \texttt{object_t} is just $N$
Unlike other quadric surfaces, there is only a single point at which a ray intercepts a plane. Therefore unlike equations we will see later, this one is not quadratic. There are some special cases we must consider:

(1) \((N \cdot D) = 0\) In this case the direction of the ray is perpendicular to the normal to the plane. This means the ray is parallel to the plane. Either the ray lies in the plane or misses the plane entirely. We will always consider this case a miss and return -1. Attempting to divide by 0 will cause your program to either fault and die or return a meaningless value.

(2) \(t_h < 0\) In this case the hit lies behind the viewpoint rather than in the direction of the screen. This should also be considered a miss and -1 should be returned.

(3) The hit lies on the viewpoint side of the screen.

\[ H = (h_x, h_y, h_z) \text{ if } h_z > 0 \text{ the hit is on the wrong side} \]

and -1 should be returned.
Testing the object, material and plane modules

At this point, it is useful to extend our material tester to include loading and testing a plane object. Each time we add a new component to the project we want to ensure that we haven't broken any existing component. This process is called regression testing.

We will use the following input. The first plane is sometimes called a "back wall". All of its points have a z-coordinate of -7 and its normal points directly outward in the +z direction. The second plane is called a "floor" but a normal floor would have a normal of 0 1 0. This "floor" is "tilted" down toward the viewer. Elements of the floor with y coordinates > 0.0 will be hidden behind the backwall.

A side view of the situation is shown below.

```plaintext
material green
{
    ambient 0 5 0
}

material brown
{
    ambient 3 3 0
}

plane wall
{
    material green
    normal 0 0 1
    point 0 0 -7
}

plane floor
{
    material brown
    normal 0 1 1
    point 0 0 -7
}
```
int main()
{
    model_t    mod;
    model_t    *model = &mod;
    material_t *mat;
    object_t   *obj1;
    object_t   *obj2;
    char       entity[16];
    int        count;

    /* Create lists */

    model->mats = list_init();
    model->objs = list_init();

    /* Load the two material definitions */

    count = fscanf(stdin, "%s", entity);
    material_init(stdin, model, 0);
    mat = (material_t *)list_get_entity(model->mats);
    assert(mat->cookie == MAT_COOKIE);
    fprintf(stderr, "loaded %s \n", mat->name);

    count = fscanf(stdin, "%s", entity);
    material_init(stdin, model, 0);
    mat = (material_t *)list_get_entity(model->mats);
    assert(mat->cookie == MAT_COOKIE);
    fprintf(stderr, "loaded %s \n", mat->name);

    /* Verify that worked */

    material_list_print(model, stderr);
/* Now load the two object definitions */

count = fscanf(stdin, "%s", entity);
plane_init(stdin, model, 0);
obj1 = (object_t *)list_get_entity(model->objs);
assert(obj1->cookie == OBJ_COOKIE);
fprintf(stderr, "loaded %s \n", obj1->obj_name);

object_list_print(model, stderr);

count = fscanf(stdin, "%s", entity);
plane_init(stdin, model, 0);
obj2 = (object_t *)list_get_entity(model->objs);
assert(obj2->cookie == OBJ_COOKIE);
fprintf(stderr, "loaded %s \n", obj2->obj_name);

/* Verify that worked */

object_list_print(model, stderr);
/* Now test the hits functions */

vec_t view = {4.0, 3.0, 5.0};
vec_t dir = {0.0, 0.0, -1.0};
double dist = 0.0;
vec_t unit;
vec_unit(dir, unit);
memset(obj1->hits, 0, sizeof(vec_t));
dist = obj1->hits(obj1, view, unit);

fprintf(stderr, "dist to plane 1 \%8.3lf \n", dist);
vec_print(stderr, "hit point", obj1->last_hit);
vec_unit(dir, unit);
memset(obj2->hits, 0, sizeof(vec_t));
dist = obj2->hits(obj2, view, unit);

fprintf(stderr, "dist to plane 2 \%8.3lf \n", dist);
vec_print(stderr, "hit point", obj2->last_hit);

/* Make sure we dont get a hit in +z space */

dir[Y] = -2.1;
vec_unit(dir, unit);
memset(obj2->hits, 0, sizeof(vec_t));
dist = obj2->hits(obj2, view, unit);

fprintf(stderr, "positive z test \n");
fprintf(stderr, "dist to plane 2 \%8.3lf \n", dist);
vec_print(stderr, "hit point", obj2->last_hit);

/* Make sure we don't get a hit on a miss... Shoot */
/* straight down at backwall */

dir[Z] = 0;
vec_unit(dir, unit);
memset(obj1->hits, 0, sizeof(vec_t));
dist = obj1->hits(obj1, view, unit);

fprintf(stderr, "vertical ray test \n");
fprintf(stderr, "dist to plane 1 \%8.3lf \n", dist);
vec_print(stderr, "hit point", obj1->last_hit);

return(0);
Program output

loaded green
loaded brown
material green
ambient 0.0 5.0 0.0
diffuse 0.0 0.0 0.0
specular 0.0 0.0 0.0

material brown
ambient 3.0 3.0 0.0
diffuse 0.0 0.0 0.0
specular 0.0 0.0 0.0

loaded wall

plane wall
material green
normal 0.0 0.0 1.0
point 0.0 0.0 -7.0
loaded floor

plane wall
material green
normal 0.0 0.0 1.0
point 0.0 0.0 -7.0
plane floor
material brown
normal 0.0 1.0 1.0
point 0.0 0.0 -7.0

dist to plane 1 12.000
hit point 4.000 3.000 -7.000
dist to plane 2 15.000
hit point 4.000 3.000 -10.000
positive z test
dist to plane 2 -1.000
hit point 4.000 -7.161 0.161
vertical ray test
dist to plane 1 -1.000
hit point 4.000 3.000 -7.000
The sphere.c module

A sphere in 3-D space is defined by:

- The location of its center, a vec_t.
- The radius of the sphere which is a scalar (double) value, r.

```c
typedef struct sphere_type
{
    vec_t      center;
    double     radius;
    vec_t      scale;    // (1, 1, 1) for spheres
} sphere_t;
```

`sphere_init()`

The `sphere_init()` function performs the same functions as `plane_init`.

```c
void sphere_init(
    FILE    *in,
    model_t *model,
    int     attrmax)  // maximum number of attributes
{
    sphere_t  *sph;
    object_t  *obj;
    int       count;
    
}
```
sphere_load_attributes -

This function works just like your other attribute loaders. It must return the number of attributes loaded,

```c
int sphere_load_attributes(
    FILE     *in,
    sphere_t *sph)
{
    ----- like plane_load_attributes -----
}
```

sphere_print

Each object specific printer is responsible for first calling its parent's print function.

```c
void sphere_print(
    object_t *obj,
    FILE     *out)
{
    ---- like plane_print
}
```

Output should look like

```
sphere       bigball         <- printed by object_print()
material     green
center         1.0   1.0  -2.0 <- printed by plane_print()
radius         0.4
```
Determining if a ray hits a sphere.

sphere_hits() -

The sphere_hits() function determine if a ray hits a sphere and if so fills in the coordinates of the hitpoint and the normal in the object_t structure.

double sphere_hits(
    object_t *obj,
    vec_t base,      /* ray base */
    vec_t dir)       /* unit direction vector */
{
}

Assume the following:

\[ V = \text{viewpoint or start of the ray} \]

\[ D = \text{a unit vector in the direction the ray is traveling} \]

\[ C = \text{center of the sphere} \]

\[ r = \text{radius of the sphere.} \]

The arithmetic is much simpler if the center of the sphere is at the origin. So we start by moving it there! To do so we must make a compensating adjustment to the base of the ray.

\[ C' = C - C = (0, 0, 0) = \text{new center of sphere} \]

\[ V' = V - C = \text{new base of ray} \]

\[ D \text{ does not change} \]
A point $P$ on the sphere whose center is $(0, 0, 0)$ necessarily satisfies the following equation:

$$p_x^2 + p_y^2 + p_z^2 = r^2 \quad (1)$$

All points on the ray may be expressed in the form

$$P = V' + t D = (v'_x + td_x, \ v'_y + td_y, \ v'_z + td_z) \quad (2)$$

where $t$ is the Euclidean distance from $V'$ to $P$

Thus we need to find a value of $t$ which yields a point that satisfies the two equations. To do that we take the $(x, y, z)$ coordinates from equation (2) and plug them into equation (1). We will show that this leads to a quadratic equation in $t$ which can be solved via the quadratic formula.

$$(v'_x + td_x)^2 + (v'_y + td_y)^2 + (v'_z + td_z)^2 = r^2$$
Expanding the expression

\[(v'_x + td_x)^2 + (v'_y + td_y)^2 + (v'_z + td_z)^2 = r^2\]

by squaring the three binomials yields:

\[(v'_x^2 + 2tv'_x d_x + t^2 d_x^2) + (v'_y^2 + 2tv'_y d_y + t^2 d_y^2) + (v'_z^2 + 2tv'_z d_z + t^2 d_z^2) = r^2\]

Next we collect the terms associated with common powers of \(t\)

\[(v'_x^2 + v'_y^2 + v'_z^2) + 2t (v'_x d_x + v'_y d_y + v'_z d_z) + t^2(d_x^2 + d_y^2 + d_z^2) = r^2\]

Now we reorder terms as decreasing powers of \(t\) and note that all three of the parenthesized trinomials represent dot products.

\[(D \cdot D)t^2 + 2(V' \cdot D) t + V' \cdot V' - r^2 = 0\]

We now make the notational changes:

\[a = D \cdot D\]
\[b = 2(V' \cdot D)\]
\[c = V' \cdot V' - r^2\]

to obtain the following equation

\[at^2 + bt + c = 0\]

whose solution is the standard form of the quadratic formula:

\[t_h = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]
Recall that quadratic equations may have 0, 1, or 2 real roots depending upon whether the *discriminant*:

\[(b^2 - 4ac)\]

is negative, zero, or positive. These three cases have the following physical implications:

- **negative** \(\Rightarrow\) ray doesn't hit the sphere
- **zero** \(\Rightarrow\) ray is tangent to the sphere hitting it at one point
  (we will consider this a miss).
- **positive** \(\Rightarrow\) ray does hit the sphere and would pass through its interior
  (this is the *only* case we consider a *hit*).

Furthermore, the two values of \(t\) are the distances from the base of the ray to the points(s) of contact with the sphere. We always seek the *smaller* of the two values since we seek to find the “entry wound” not the “exit wound”.

Therefore, the \(\text{hits\_sphere}()\) function should return

\[
\begin{align*}
    t_h & = -b - \sqrt{b^2 - 4ac} \\
    & \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]

if the discriminant is positive and

\[
    t_h = -1
\]

otherwise.
Determining the coordinates of the hit point on a sphere.

The \((x, y, z)\) coordinates are computed as follows.

\[
H = V + t_h D
\]

where \(t_h\) is the smaller root of the quadratic equation on the previous page.

Important items to note are:

- The actual base of the ray \(V\) and not the translated base \(V'\) must be used.
- The vector \(D\) must be a unit vector in the direction of the ray.

Determining the surface normal at the hit point.

The normal at any point \(P\) on the surface of a sphere is a vector from the \textit{center} to the \textit{point}. Thus

\[
N = P - C \quad (\text{note that } N \text{ will be a unit vector } \iff r = 1)
\]

Therefore a unit normal may be constructed as follows:

\[
N_u = (H - C) / ||(H - C)||
\]
The main function

A properly designed and constructed program is necessarily modular in nature. Modularity is somewhat automatically enforced in O-O languages, but new C programmers often revert to an ugly pack-it-all-into-one-main-function approach.

To discourage this in the raytracing program, deductions will be made for:

1 - Functions that are too long (greater than 30 lines)
2 - Nesting of code greater than 2 deep (NO nested loops)
3 - Exception to 2: Its OK to have an if inside a loop.
4 - Lines that are too long (greater than 72 characters)
The `main()` function

Here is the main function for the final version of the ray tracer.

```c
/* main.c */

#include "ray.h"

int main(
    int argc,
    char *argv[])
{
    model_t *model;

    /* Load and dump the model */
    model = model_init(stdin);
    model_print(model, stderr);

    /* Raytrace the image */
    image_create(model);
    return(0);
}
```
Loading the model description

For the scene specification

- *material, object, and light* data may be intermixed
- the *camera* definition may appear anywhere
- *materials* must be defined *before being referenced* in an object definition
- groups of attributes must appear in the top-down order in which the *init* function is called.
- attributes processed within a specific *init* function may appear in any order.
- missing attributes must be set to zero.

```plaintext
camera cam1
{
    pixeldim  640 480
    worlddim  8 6
    viewpoint  4 3 6
}

material green
{
    diffuse 0 1 0
    ambient 0 5 0
}

plane leftwall
{
    material green
    normal 3 0 0.2
    point  0 0 0
}

plane rightwall
{
    material green
    point   8 0 0
    normal -3 0 0.2
}
```
The `model_init()` function

This function be contained in the source module `model.c`. It controls the loading of camera, material, object, and light definitions. When it completes, the cam pointer must point to a complete camera_t structure and material, object and light definitions specified in the input file will have been read, and for each material, object or light definition in the file a new structure of type material_t, object_t, or light_t must reside on the appropriate list. (If the input file contains the definition of three planes, three object_t’s must be on the objs list. )

typedef struct model_type
{
    camera_t *cam;   // The camera structure
    list_t *mats;   // The head of the material list
    list_t *objs;   // The head of the visible obj list
    list_t *lgts;   // The head of the light list
} model_t;
Implementing the `model_init_function`

Specifically it must: `malloc()` the `model_t` structure and set it to zero, call `list_init()` three times to set up the `material`, `object`, and `light` lists, call an internal `model_load_entities()` function to load the model data, and then return the address of the `model_t` structure.

```c
/**/
/* Init model data */

model_t *model_init(
    FILE *in)
{
    model_t *model = malloc(sizeof(model_t));
    assert(model != NULL);
    memset(model, 0, sizeof(model_t));

    /* Create and initialize material structure list */
    model->mats = list_init();
    assert(model->mats != NULL);

    /* Create and initialize visible object structure list */

    /* Create and initialize light structure list */

    /* read in the camera, materials, objects, lights */
    model_load_entities(in, model);
    return(model);
}
```

Step one is to `create` the lists. This step READS NO INPUT DATA.

Step two is to `read and store` the model data.
The model_load_entities() function

The model loader operates in a way similar to the attribute loaders that you have already written.

camera cam1
{
  pixeldim  640 480
  worlddim  8 6
  viewpoint  4 3 6
}

material green
{
  diffuse 0 1 0
  ambient 0 5 0
}

plane leftwall
{
  normal 3 0 0.2
  point  0 0 0
  material green
}

plane rightwall
{
  material green
  point  8 0 0
  normal -3 0 0.2
}
The model_load_entities() function

static void model_load_entities(
    FILE    *in,
    model_t  *model)
{
    char entityname[NAME_LEN];
    int    count;

    memset(entityname, 0, sizeof(entityname));

    /* Here entityname should be one of "material", */
    /* "light", "plane", "camera", etc */

    count = fscanf(in, "%s", entityname);
    while (count == 1)
    {
        process one entity and read next entity name
    }
}
The *model_print* function

This function just drives the process of producing a nicely formatted version of the contents of the material list, the object list, and the light list. The entity-type specific functions shown are responsible for the details.

```c
/**
 * dump model data */

void model_print(
    model_t *model,
    FILE    *out)
{
    // Invoke specific print routines
}
```
Avoiding parsing the input file altogether

In building programs it's often useful to employ temporary skeletal modules that facilitate the building and testing of other components but are ultimately thrown away at the end of the project. For example, if this were a team project it would be desirable for the ray tracing team to press on in parallel with the parsing team's activities instead of having to wait on a functional parser.

We can view this exercise as transforming or model specification language to C! In fact the C version looks quite similar to the target language.

```c
/* modelstat.c */

/* This module provides a statically defined model that can */
/* be used to test the raytracing system. */

#include "ray.h"

material_t mat1 =
{
    cookie: MAT_COOKIE,
    name: "green",
    ambient: {0, 5, 0},
};

material_t mat2 =
{
    cookie: MAT_COOKIE,
    name: "yellow",
    ambient: {6, 5, 0},
};

material_t mat3 =
{
    cookie: MAT_COOKIE,
    name: "gray",
    ambient: {4, 4, 4},
};
```
Now we define the plane structures. Again the definitions are quite consistent with the target language.

```cpp
plane_t plane1 =
{  
    normal: {3, 0, 1},
    point:  {0, 0, 0},
};

plane_t plane2 =
{  
    normal: {-3, 0, 1},
    point:  {8, 0, 0},
};

plane_t plane3 =
{  
    normal: {0, 1, 0},
    point:  {0, 0, 0},
};
```
The object structure definitions combine elements of the original input language, but more reflect the actions of the program.

```c
object_t object1 =
{
   cookie:   OBJ_COOKIE,
   obj_name:  "leftwall",
   hits:     plane_hits,
   priv:     (void *)&plane1,
   mat:      &mat1,
};

object_t object2 =
{
   cookie:   OBJ_COOKIE,
   obj_name:  "rightwall",
   hits:     plane_hits,
   priv:     (void *)&plane2,
   mat:      &mat2,
};

object_t object3 =
{
   cookie:   OBJ_COOKIE,
   obj_name:  "floor",
   hits:     plane_hits,
   priv:     (void *)&plane3,
   mat:      &mat3,
};
```
Linking the model together.

The last (and ugliest) piece of the puzzle is to handcraft the object list and put it in the model structure. Note that the *material* list is not necessary because there is on material dumper and no material find needed.

```c
link_t link1 =
{
next: NULL,
entity: (void *)&object2,
};

link_t link2 =
{
next: &link1,
entity: (void *)&object1,
};

link_t link3 =
{
next: &link2,
entity: (void *)&object3,
};

list_t list1 =
{
first: &link3,
last: &link1,
current: &link1,
};

model_t model =
{
objs: &list1,
};

model_t *model_init(
FILE *in)
{
return(&model);
};
```
Creating an image

We continue to strive to build simple easy to grasp components! Obviously, this could be done by massively nesting loops and building functions 100+ lines long. Even some faculty and professional programmers will do it this way.

But if you do it my way you avoid the possibility that one day you will be recreated as a VooDoo doll by those charged with trying to understand and maintain what you wrote!!

/**
/* This function is the driver for the raytracing procedure */

void image_create(
model_t  *model)
{
    int y;
    camera_t  *cam = model->cam;

    /* Fire ray(s) through each pixel in the window */
    for (y = 0; y < cam->pixel_dim[1]; y++)
    {
        make_row(model, y);
    }

    /* Ask camera_write_image to ppm image */
    camera_write_image(model->cam, stdout);
}

Processing a row of pixels

The most common way to mess this up is to forget which element of the `pixel_dim` array represents the horizontal size and which represents the vertical size.

```c
static inline void make_row(
    model_t *model,
    int      y)
{
    int   x;
    camera_t *cam = model->cam;

    for (x = 0; x < cam->pixel_dim[0]; x++)
    {
        make_pixel(model, x, y);
    }
}
```
Building a pixel

This function is called for each pixel in the image. Eventually we will try to minimize the “jaggies” by building a loop in here in which we randomize the ray direction and average the computed pixel values.

```c
static inline void make_pixel(
    model_t *model,
    int      x,
    int      y)
{
    vec_t  raydir;
    drgb_t d_pix = {0.0, 0.0, 0.0};
    camera_t  *cam = model->cam;
    int    i;

    /* This function was written previously */

    camera_getdir(cam, x, y, raydir);

    #ifdef DBG_PIX
    fprintf(stderr, "PIX %4d %4d - ", y, x);
    #endif

    /* The ray_trace function determines the pixel color in */
    /* d_rgb units.. The last two parameters are used ONLY  */
    /* in the case of specular (bouncing) rays which we are */
    /* not doing yet.                                       */

    ray_trace(model, cam->view_point,
                raydir, d_pix, 0.0, NULL);

    /* This function must convert the pixel value from drgb_t */
    /* [0.0, 1.0] to irgb_t (0, 255) and to store it in the  */
    /* “upside down” location in the pixmap                   */

    camera_store_pixel(cam, x, y, d_pix);

    return;
}
```
The ray_trace() function

/**
/* This function traces a single ray and returns the */
/* composite intensity of the light it encounters */

void ray_trace(
model_t  *model,
vec_t     base,        /* location of viewer or previous hit */
vec_t     dir,         /* unit vector in direction of object */
drgb_t    dpix,        /* pixel return location */
double    total_dist,   /* distance ray has traveled so far */
object_t *last_hit)    /* most recently hit object */
{
    object_t *closest;
    double   mindist;
    drgb_t   thisray = {0.0, 0.0, 0.0};

    Ask find_closest_object() to set the closest pointer
    If it returns an object pointer
    {
        #ifdef DBG_HIT
            fprintf(stderr, "%-12s HIT: (%5.1lf, %5.1lf, %5.1lf)",
                closest->obj_name,
                closest->last_hit[X],
                closest->last_hit[Y],
                closest->last_hit[Z]);
        #endif

        use the objects polymorphic closest->ambient() function copy the object's ambient
        reflectivity to “thisray”
    }
    scale the values of “thisray” by 1 / distance to the closest object
    add the value of “thisray” to pix
        #ifdef DBG_DRGB
            fprintf(stderr, "%-12s DRGB: (%5.2lf, %5.2lf, %5.2lf)",
                closest->objname, dpix[R], dpix[G], dpix[B]);
        #endif
    }
}
Debugging output

The raytracer is sufficiently complicated that debugging output may be required for problem resolution. The C-compiler preprocessor `cpp` permits us to conditionally compile or not compile statements into a program. If we include the line:

```
#define DBG_DRGB
```

in the source code then this statement will be compiled and produce debugging output. But if we comment it out the debug output will not be produced.

```
#ifdef DBG_DRGB
   fprintf(stderr, "%-12s DRGB: (%5.2lf, %5.2lf, %5.2lf)",
            closest->objname, pix->r, pix->g, pix->b);
#endif
```

Instead of having to comment in/out the definition, the C compiler allows you to define a symbol on the command line:

```
gcc -c -g -DDBG_DRGB raytrace.c
```
A makefile for a multi-module program:

The Unix *make* program is a handy utility that can be used to build things ranging from programs to documents. Elements of significance include:

- **targets**: labels that appear in column 1 and are followed by a the character "\: ". The *make* command can take a target as an operand as in *make ray*.
- **dependencies**: are files that are enumerated following the name of the target. If any dependency is newer than the target, the target will be rebuilt.
- **rules**: are specified in lines following the target and specify the procedure for building the target. Rules *must* start with a *tab character*. In the example below the tab has been expanded as spaces *but you may not enter spaces*.

The following *makefile* can be used build the executable ray tracer named *ray* (assuming that it requires only the .o files enumerated in the command).

```make
a.out: main1.o model.o camera.o list.o material.o plane.o \
    object.o sphere.o \ 
    vector.h ray.h rayfuns.h rayhdrs.h 
    gcc -Wall -g *.o -lm

.c.o: $<
    -gcc -c -Wall -c -g $<  2> $(@:.o=.err)
    cat $*.err
```

The target .c.o: is called a *suffix rule*. It is telling *make* to use the commands that follow whenever it needs to make a .o file from a .c file.

There are a number of predefined macro based names:

- `$@` -- the current target's full name
- `$?` -- *a list of the target's changed dependencies*
- `$<$ -- similar to `$?` but identifies a single file dependency and is used only in suffix rules
- `$*` -- the target file's name without a suffix

Another handy macro based facility permits one to change prefixes on the fly. The macro `$(@:.o=.err)` says use the target name but change the .o to .err.

The same result effect may be obtained using `$* .err` as is done in the subsequent *cat* command.
Using user written macros in makefiles

The makefile on the previous page is actually broken! All of the .c files depend on ray.h and should be recompiled if ray.h changes, but this will not happen! We could fix this by typing in a collection of other dependencies but the macro facility simplifies that.

Make macros are similar in spirit to Unix environment variables. In fact environment variables can be accessed in make files via macro calls. However, it is typically the case that the macros are defined within the makefile. Here is a makefile that is used to build a complete raytracer. A macro is defined by using the syntax MACRO-NAME = macro value. Many people use the convention of making names all capital but that is not required.

All of the .o files necessary to build in are defined using the macro name RAYOBS. The \ character at the end of all but the last line is the standard Unix contuation character. The # character at the start of a line turns the line into a comment.

A macro is invoked using the syntax $(MACRO-NAME). The result of the invocation is that the string $(MACRO-NAME) is replaced by the current value of the macro.

RAYOBS = main1.o model.o camera.o list.o material.o plane.o \  object.o sphere.o
RAYHDRS = vector.h ray.h rayfuns.h rayhdrs.h
a.out: $(RAYOBS)
gcc -Wall -g *.o -lm
$(RAYOBS): $(RAYHDRS) makefile
.o: $<
    -gcc -c -Wall -c -g $< 2> $(@:.o=.err)
cat $*.err
Defining debug control symbols in the makefile

Use the CFLAGS macro to enable precisely those debug aids that you need:

```
CFLAGS = -DDBG_PIX -DDBG_HIT
```

```
ray: $(RAYOBJS)
    gcc -Wall -o ray -g $(RAYOBJOJS) -lm

$(RAYOBJOJS): $(INCLUDE) makefile
    .c.o: $<
        -gcc -c -Wall $(CFLAGS) -c -g $<  2> $(@:.o=.err)
        cat $*.err
```

This code is in the `make_pixel()` function:

```
#ifdef DBG_PIX
    fprintf(stderr, "\nPIX %4d %4d - ", y, x);
#endif
```

This code is in the `ray_trace()` function.

```
#ifdef DBG_HIT
    fprintf(stderr, "%-12s HIT:(%5.1lf, %5.1lf, %5.1lf)",
            closest->objname,
            closest->hitloc.x, closest->hitloc.y,
            closest->hitloc.z);
#endif
```
Because of how the \n characters are used in the format string they work together to produce this useful output.

```
PIX   21   16 - leftwall     (  1.3,   1.4,  -4.0)
PIX   22   16 - leftwall     (  1.5,   1.3,  -4.4)
PIX   23   16 - leftwall     (  1.6,   1.2,  -4.8)
PIX   24   16 - leftwall     (  1.8,   1.0,  -5.3)
PIX   25   16 - leftwall     (  2.0,   0.8,  -5.9)
PIX   26   16 - leftwall     (  2.2,   0.6,  -6.5)
PIX   27   16 - leftwall     (  2.4,   0.4,  -7.2)
PIX   28   16 - leftwall     (  2.7,   0.2,  -8.0)
PIX   29   16 - floor        (  3.0,   0.0,  -8.5)
PIX   30   16 - floor        (  3.4,   0.0,  -8.5)
PIX   31   16 - floor        (  3.8,   0.0,  -8.5)
PIX   32   16 - floor        (  4.2,   0.0,  -8.5)
PIX   33   16 - floor        (  4.6,   0.0,  -8.5)
PIX   34   16 - floor        (  5.0,   0.0,  -8.5)
PIX   35   16 - rightwall    (  5.3,   0.2,  -8.0)
PIX   36   16 - rightwall    (  5.6,   0.4,  -7.2)
PIX   37   16 - rightwall    (  5.8,   0.6,  -6.5)
PIX   38   16 - rightwall    (  6.0,   0.8,  -5.9)
PIX   39   16 - rightwall    (  6.2,   1.0,  -5.3)
PIX   40   16 - rightwall    (  6.4,   1.2,  -4.8)
```
Polymorphism - II:

We have already discussed how the \textit{hits} and \textit{printer} functions provide polymorphic behavior in the object "class". Each specialization (plane, sphere) \textbf{must provide} its own characteristic functions because the default functions perform no useful function. Their only use is to prevent an instant segfault when the object is to be tested for a ray intersection.

In other cases most specializations \textit{may want to use the default} function. For example, the default \texttt{ambient} function simply calls \texttt{material\_getamb()}. Recall that in the ambient only raytracer the last steps of the operation are:

\begin{verbatim}
    add mindist to total_dist
    set intensity to the ambient reflectivity of closest object
    divide intensity by total_dist
\end{verbatim}

The first inclination is to implement the small amount of code in step 2 in the obvious way:

\begin{verbatim}
    this_pix[R] = closest->mat->ambient[R];
    this_pix[G] = closest->mat->ambient[G];
    this_pix[B] = closest->mat->ambient[B];
    or
    pix_copy(closest->mat->ambient, this_pix);
\end{verbatim}

However that approach would make it \textit{not easy to override} the \texttt{default} behavior. Thus a better approach is to replace the three lines above by:

\begin{verbatim}
    closest->ambient(closest->mat, this_pix);
\end{verbatim}

During the \texttt{object\_init()} object constructor sets the \texttt{ambient()} function pointer to the “\texttt{material\_getamb()}” function which contains the three lines of code we just replaced.

While this adds a slight bit of run time overhead, it also provides us with an easy hook with which we may override the \texttt{material\_getamb()} with a custom routine. We will see an example of this with the tiled plane object.
Tiled planes

The tiled plane object is commonly found in ray tracing models.

We will use it to demonstrate how the inheritance hierarchy of specialization can be extended. In Object Oriented terminology the \textit{object}_t structure is called a \textit{base class}. The base class is at the top or root of a class hierarchy. The \textit{plane}_t and \textit{sphere}_t are \textit{specializations} of the \textit{object}_t and are called \textit{derived classes}. The tiled plane \textit{tplane}_t and the procedural plane \textit{pplane}_t are further specializations of the \textit{plane}_t.
The tiled_plane model object description

The tiled plane requires two attributes beyond those of the regular plane.

The dimensions specify the size of the tiling in world coordinates
The altmaterial specifies the color of the alternate tiles.

tiled_plane  floor
  {
    material white
    normal  0 1 0
    point 0 0 0
    dimension 2.0 3.0
    altmaterial green
  }

A specialization always requires that some aspect of the behavior of the parent class be modified. If no modifications at all were required, we could just use the base class.

A specialization never requires that all aspects of the parent's behavior be overridden. In that case it would be appropriate to create a new class.

When a new tiled_plane is created three structures must be allocated and linked together:

- The object_init() function is called by plane_init(). It mallocs the object_t.
- The plane_init() function is called by tplane_init(). It mallocs the plane_t and sets the priv pointer in the object_t.
- The tplane_init() function is called by the model loader. It mallocs the tplane_t and sets the priv pointer in the plane_t.
The *tplane_t* structure and the *tplane_init()* function.

The structure is shown below and it must be filled in by *tplane_init()* from the input data

```c
/* Tiled plane... descendant of plane */
typedef struct tplane_type
{
    char    matname[NAME_LEN];
    material_t *background; /* background color */
    double   dimension[2];  /* dimension of tiles */
}  tplane_t;
```

The dimension parameter specifies the (x, z) dimensions of a single tile in world coordinates. The *altmaterial* is the name of the material used in alternate tiles. The *tplane_init()* function must ask *material_search()* to lookup the corresponding material_t.

```c
tiled_plane floor
{
    material white
    normal   0 1 0
    point    0 0 0
    dimension 2.0 3.0
    altmaterial green
}
```
The `tplane_init()` function

```c
object_t *tplane_init(
    FILE *in,
    model_t *model)
```

The function is called from the model loaders. Its missions are to:

- invoke `plane_init(in, model, 2)` to create the `plane_t` and `object_t` structures
- The 2 is passed to `plane_init()` to tell it not to process more than two attributes. You will need to fix `plane_load_attributes()` to make it honor this limit.
- recover a pointer from to the object structure from the model->objs list
- recover a pointer to the plane structure from the `priv` pointer of the object
- malloc a `tplane_t` and set the `priv` pointer in the `plane_t` structure
- parse the attributed data
- set the `ambient` pointer in the `object_t` to point to the `tplane_ambient()` function
- set the `printer` pointer in the `object_t` to point to `tplane_print`
- set `obj_type` field in the `object_t` to "tiled plane".

The `tplane_init()` function need not override the hits function provided `plane_init()`.

The `tplane_print()` function

```c
static void tp_print(
    FILE *out,
    object_t *obj)
```

- invoke the `plane_print()` function
- print the `tplane` attributes
The `tplane_ambient()` function

This is the function that actually gives the `tplane` its characteristic behavior.

```c
void tplane_ambient(
    object_t *obj,
    drgb_t value)
{
    int foreground = tplane_foreground(obj);

    if (foreground)
        material_getamb(obj, obj->mat, value);
    else
        copy ambient reflectivity from background material
}
```
The `tplane_foreground()` function

```c
int tplane_foreground(
    object_t *obj)
{
    Compute x_ndx = tile index of the hitpoint in the x direction
    Compute z_ndx = tile index of the hitpoint in the z direction

    if ((x_ndx + z_ndx) is an even number)
        return(1);
    else
        return(0);
}
```

A tile index is an int It is computed by adding 10,000 to the x (or z) coordinate of the hitpoint and dividing by the x (or z) dimension of the tile.
Procedural surfaces

Procedural surfaces are those in which an object's reflectivity properties are *modulated* as a function of the location of the hit point on the surface of the object.

There are literally an infinite number of ways to do this. In the next few pages we propose a framework for incorporating procedurally shaded surfaces into raytraced images.
Implementation of procedural shaders

Construction of such shaders is facilitated by the use of both inheritance and polymorphism within a C language framework. The procedurally shaded plane is a lightweight refinement of the plane_t.

typedef struct pplane_type
{
   int     shader;
}  pplane_t;

The distinction between a standard plane and a procedurally shaded plane is made at object initialization time by the pplane_init() function when it establishes a single function pointer (for ambient only images) that provides the polymorphic behavior.

That function pointer is taken from a table of pointers to programmer provided functions are contained in the module pplane.c and perform the procedural shading. These procedural shading functions are passed pointers to the object_t structure and to the dgrb_t intensity vector whose (r, g, b) components are filled procedurally. Here is an example in which there are three possible shaders.

static void (*pplane_shaders[])(object_t *obj, material_t *mat, dgrb_t *value) =
{
   pplane0_ambient,
   pplane1_ambient,
   pplane2_ambient,
};

#define NUM_SHADERS sizeof(pplane_shaders)/sizeof(void *)

Note that:

1. The number of elements in the array is not explicitly specified.
2. The value NUM_SHADERS can be computed by dividing the size of the table by the size of a single pointer.
The index of the shader to be used is supplied in the model description as shown below.

```plaintext
pplane floor
{
    material gray
    normal 0 1 0
    point 0 0 -8
    shader 0
}

pplane backwall
{
    material gray
    normal 0 0 1
    point 4 3 -8
    shader 1
}
```
The `pplane_init()` function

As shown below the `pplane_init()` function simply invokes the `plane_init()` function to construct the object and then overrides the default `getamb()` function, replacing it with the shader function whose index is provided in the model description files.

```c
/**/
object_t *pplane_init(
FILE    *in,
model_t *model,
int  attrmax)
{
    pplane_t *ppln;
    plane_t  *pln;
    object_t *obj;

    int   mask;

    plane_init(in, model, 2);

    /* Recover object pointer from object list */
    /* Recover plane pointer from object pointer */

    /* malloc a pplane structure and link plane structure to it */

    count = pplane_load_attributes(in, ppln);
    assert(count = 1);

    strcpy(obj->objtype, "pplane");
    obj->ambient = pplane_shaders[ppln->shader];
    obj->printer = pplane_print;
}
```

The mysterious `attrmax` parameter finally gets to do something!
Tiled shading

To produce a tiled “floor” the modulation must be a function of the $x$ and $z$ coordinates because the $y$ coordinate does not vary on the floor. For a “backwall” it would be necessary to modulate $x$ and $y$, and for a “sidewall” it would be $y$ and $z$.

```c
void pplane0_amb(
  object_t *obj,
  drgb_t *value)
{
  int    ix;
  int    iz;

  ix = 2  * obj->hitloc.x  + 1000;
  iz = 2  * obj->hitloc.z  + 1000;
  pix_copy(&obj->mat->ambient, value);

  if ((iz + ix ) & 1)  // test for odd or even sum
  {
    value->r = 0.0;  // make pixel cyan
  }
  else
  {
    value->b = 0.0;  // make pixel yellow
  }
}
```

The factor of 2 controls the width of the tiles. The larger the factor the smaller the tile. The value of 1000 is known as Westall's hack for preventing an ugly double wide strip at the origin.
**Continuously modulated shading**

The image shown below is produced by a procedural shader that continuously modulates the ambient reflectivity.

![Image](image.png)

The modulation function is shown below. A vector \( V \) in the direction from the point defining the plane location to the hitpoint is computed first. Then the angle that the vector makes with the positive \( X \) axis is computed. Finally the red, green and blue components are modulated using the function

\[
1 + \cos(w \cdot t + f)
\]

where the angular frequency \( w \) is 2 for all three colors, and phase angles \( f \) are 0, \( \frac{2\pi}{3} \), and \( \frac{4\pi}{3} \) respectively. Different effects may be obtained by using different frequencies and phase angles for each color, and it is also possible to combine continuous modulation with striping or tiling.

```c
vec_diff(&p->point, &obj->hitloc, &vec);

v1 = (vec.x / sqrt(vec.x * vec.x + vec.y * vec.y));
t1 = acos(v1);

if (vec.y < 0)
t1 = 2 * M_PI - t1;

value->r = 6 * (1 + cos(2 * t1));
value->g = 6 * (1 + cos(2 * t1 + 2 * M_PI / 3));
value->b = 6 * (1 + cos(2 * t1 + 4 * M_PI / 3));
}
```
The cross product of two vectors

Given two linearly independent (not parallel) vectors:

\[ V = (v_x, v_y, v_z) \]
\[ W = (w_x, w_y, w_z) \]

The cross product sometimes called outer product is a vector which is orthogonal (perpendicular to) both of the original vectors.

\[ V \times W = (v_y w_z - v_z w_y, \ v_z w_x - v_x w_z, \ v_x w_y - v_y w_x) \]

\[ (1, 1, 1) \times (0, -1, 0) = (1, 0, -1) \]

Notes:

The vector (0, -1, 0) is the negative y axis. Therefore, any vector that is perpendicular to it must lie in the \( y = 0 \) plane. The projection of the vector (1, 1, 1) onto the \( y = 0 \) plane is the vector (1, 0, 1). The vector (1, 0, -1) is then perpendicular to this vector and lies in the \( y=0 \) plane.

In a right-handed coordinate system

\[ X \times Y = Z \]
\[ Y \times Z = X \]
\[ Z \times X = Y \]

Right thumb (x-axis) x forefinger (y-axis) = middle finger (z-axis).
The `vec_cross()` function

You will add the following function to your `vector.h` collection.

```c
/**
/* Compute the outer product of two input vectors */

static inline void vec_cross(
vec_t  v1,     /* Left input vector  */
vec_t  v2,     /* Right input vector */
vec_t  v3)     /* Output vector       */
{
}
```
Projection

Assume that $V$ and $N$ are unit vectors. The projection, $Q$, of $V$ on $N$ is shown in red. It is a vector in the same direction as $N$ but having length $\cos(\theta)$. Therefore

$$Q = (N \cdot V) N$$

Now assume that $N$ is a normal to a plane shown as a yellow line. The projection, $P$, of $V$ onto the plane is shown in magenta and is given by $V + G$ where $G$ is the vector shown in green.

Since $G$ and $Q$ have have the same length but point in opposite directions, $G = -Q$

Therefore the projection of a vector $V$ onto a plane with normal $N$ is given by:

$$P = V + G = V - Q = V - (N \cdot V) N$$

or (possibly ?????)

$$\text{vec_diff}(\text{vec_scale}(\text{vec_dot}(N, V), N), V, P);$$

In building your new linear algebra routines it is desirable to build upon existing ones where possible but extreme levels of nesting of function calls as shown here can complicate debugging.

```c
/**
 * project a vector onto a plane */

static inline void vec_project(
    vec_t n,    /* plane normal */
    vec_t v,    /* input vector */
    vec_t w)    /* projected vector */
```
Reflection

Basic physics says: The angle of incidence (the angle the incoming ray makes with the normal at the hitpoint) is equal to the angle of reflection

```c
vec_reflect(
    vec_t  unitin, /* unit vector in incoming direction of the ray */
    vec_t  unitnorm, /* outward surface normal */
    vec_t  unitout); /* unit vector in outgoing direction ray */
```

Let

\[ U = -\text{unitin} \]
\[ N = \text{unitnorm} \]

Then

\[ U + V = 2N \cos(T) \] where \( T \) is the angle between \( U \) and \( N \)

\( \cos(T) = U \cdot N \)

so

\[ U + V = 2N (U \cdot N) \]

and

\[ V = 2N (U \cdot N) - U \]
Matrices

Matrix operations are useful in transforming three-dimensional coordinate systems in ways that make it easier to determine if and where an object is hit by a ray. There are alternative approaches to creating a 3 x 3 matrix.

Probably the most obvious is:

```c
double matrix[3][3];
```

we will use the following version of that:

```c
typedef double mtx_t[3][3];
mtx_t matrix;
```

By convention the first subscript refers to the row and that second the column number.

To set the element in the 3\textsuperscript{rd} column of the middle row to fifteen do,

```c
matrix[1][2]= 15.0;
```

Since each row of a matrix is a vector, we can use our vector functions to operate on rows of a matrix.

To add the first two rows of a matrix:

```c
vec_t sum;
vec_sum(matrix[0], matrix[1], sum);
```

Suppose \( \mathbf{V} \) and \( \mathbf{W} \) are vectors and \( a \) is a scalar value. A function \( F \) that maps three-dimensional space to three-dimensional space is a linear transformation if and only if

\[
F(a\mathbf{V} + \mathbf{W}) = a F(\mathbf{V}) + \mathbf{W} \quad \text{for any choice of } a, \mathbf{V}, \text{ and } \mathbf{W}
\]

Furthermore, any linear transformation may be represented by multiplication of a vector by a matrix.
Multiplication of a matrix times a vector.

The product of a 3 x 3 matrix with a 3-d column vector is a 3-d vector. The multiplication rule is as follows:

\[ product[i] = \text{the dot product of the } ith \text{ row of the matrix with the vector.} \]

\[
\begin{array}{ccc}
1.0 & 1.0 & 0.0 \\
-1.0 & 1.0 & 0.0 \\
0.0 & 0.0 & 1.0 \\
\end{array}
\times
\begin{array}{c}
1.0 \\
0.0 \\
-2.0 \\
\end{array}
= 
\begin{array}{c}
1.0 \\
-1.0 \\
-2.0 \\
\end{array}
\]

The \texttt{vec_xform()} function should multiply a vector by a matrix.

```c
static inline void vec_xform(
    mtx_t m,     /* input matrix             */
    vec_t v1,    /* vector to be transformed */
    vec_t v2)    /* output vector            */
{
    vec_t work;  /* avoid aliasing problems */

    /* Perform the transform using work for output */

    /* Copy work back to v2 */

    vec_copy(work, v2);
}
```
**The transpose of a matrix:**

The transpose of a three by three matrix is also three by the matrix. Its elements are given by a simple rule:

\[
\text{transpose}[i][j] = \text{original}[j][i]
\]

\[
\begin{bmatrix}
1.0 & 3.0 & 2.0 \\
1.0 & 2.0 & -2.0 \\
-2.0 & 0.0 & 1.0
\end{bmatrix}
\]

\[
\begin{bmatrix}
T \\
1.0 & 1.0 & -2.0 \\
3.0 & 2.0 & 0.0 \\
2.0 & -2.0 & 1.0
\end{bmatrix}
\]

Notes:

- The diagonal elements of a matrix and its transpose are identical. Off diagonal elements are interchanged in a symmetrical way.

- The transpose of a matrix is in general not the same as the inverse of a matrix.

```c
static inline void mtx_transpose(
    mtx_t m1,    /* Input matrix            */
    mtx_t m2)    /* Output transpose        */
{
    mtx_t mwork;  /* Avoid aliasing problems */

    mwork[0][0] = m1[0][0];
    mwork[0][1] = m1[1][0];
    mwork[0][2] = m1[2][0];

    etc....

    copy m3 back to m2..
}
```
Rotation matrices

Rotation matrices are used to rotate coordinate systems in 3-space. They have some special properties:

The three rows are mutually orthogonal unit vectors. That is, the dot product of any pair of rows is 0.

The three columns are also mutually orthogonal unit vectors.

The inverse of a rotation matrix is its transpose.

The 1st row of a rotation matrix is a vector which will be mapped to \([1, 0, 0]\) under the rotation. The 2nd row is a vector will be mapped to \([0, 1, 0]\) and the third row is a vector that will be mapped to \([0, 0, 1]\).

This example shows that the middle row is mapped to \((0, 1, 0)\)

\[
\begin{vmatrix}
  r_{0,0} & r_{0,1} & r_{0,2} \\
  r_{1,0} & r_{1,1} & r_{1,2} \\
  r_{2,0} & r_{2,1} & r_{2,2}
\end{vmatrix}
= \begin{vmatrix}
  r_{1,0} \\
  r_{1,1} \\
  r_{1,2}
\end{vmatrix}
= \begin{vmatrix}
  0 \\
  1 \\
  0
\end{vmatrix}
\]
Constructing rotation matrices

Suppose V and W are orthogonal unit length vectors. Suppose we want to create a rotation matrix $M$ that will rotate V into the X-axis and W into the Z-axis.

```c
vec_t V;
vec_t W;
mtx_t M;

vec_copy(V, M[0]); // V will become the X-axis
vec_copy(W, M[2]); // W will become the Z-axis

/* The middle row Y = Z x X */
vec_cross(M[2], M[0], M[1]);
```
Another example

Suppose V and W are not necessarily orthogonal unit length vectors. Suppose we want to create a rotation matrix $M$ that will rotate W into the Z-axis and force V to lie in the positive Y, Z ($X = 0$) plane.

```
vec_t  w = {1.0, 0.0, 1.0};
vec_t  v = {1.0, 1.0, 1.0};
vec_t  v3;
vec_t  v4;
mtx_t  m1;
mtx_t  m2;
```

All rows of rotation matrices must be unit vectors!

```
vec_unit(v, v);
vec_unit(w, w);
vec_print(stderr, "v ", v);
vec_print(stderr, "w ", w);
```

If we want W to end up on the +Z access and V to lie in the $X = 0$ plane, then a vector perpendicular to both W and V must end up pointing along the +X axis. Therefore, the vector $V \times W$ must be what gets mapped to the X axis. It is important to note that the cross product in not length preserving in general! Therefore we must renormalize the 1st row of the matrix.

```
vec_cross(v, w, m1[0]);
vec_unit(m1[0], m1[0]);
```

Since W is to be mapped to the positive z-axis we just copy it to the bottom row of the matrix.

```
vec_copy(w, m1[2]);
```

Since the missing middle row must be orthogonal to the other two rows it may be computed via the cross product. The order here is important! $Z \times X = Y$ but $X \times Z = -Y$!

```
vec_cross(m1[2], m1[0], m1[1]);
```
The matrix is now complete so we print it out.

```c
vec_print(stderr, "r0 ", m1[0]);
vec_print(stderr, "r1 ", m1[1]);
vec_print(stderr, "r2 ", m1[2]);
```

We now apply the matrix to V and W

```c
vec_xform(m1, v, v3);
vec_xform(m1, w, v4);
```

and then print the transformed vectors

```c
vec_print(stderr, "v3 ", v3);
vec_print(stderr, "v4 ", v4);
```

The inverse of a rotation is its transpose.

```c
mtx_transpose(m1, m2);
```

So if we apply the inverse to v3 and v4, we should get back the original V and W

```c
vec_xform(m2, v3, v3);
vec_xform(m2, v4, v4);
vec_print(stderr, "v3 ", v3);
vec_print(stderr, "v4 ", v4);
```
Normalized V and W vectors

\[
\begin{array}{ccc}
v & 0.577 & 0.577 & 0.577 \\
w & 0.707 & 0.000 & 0.707 \\
\end{array}
\]

The rotation matrix

\[
\begin{array}{ccc}
r0 & 0.707 & 0.000 & -0.707 \\
r1 & -0.000 & 1.000 & 0.000 \\
r2 & 0.707 & 0.000 & 0.707 \\
\end{array}
\]

Transformed V and W. Note that the transformed V lies in the positive Y-Z plane (its x coordinate is 0) and the transformed W is the positive Z axis.

\[
\begin{array}{ccc}
v3 & 0.000 & 0.577 & 0.816 \\
v4 & 0.000 & 0.000 & 1.000 \\
\end{array}
\]

After applying the inverse transformation, the vectors are transformed back to their original values shown at the top of this page.

\[
\begin{array}{ccc}
v3 & 0.577 & 0.577 & 0.577 \\
v4 & 0.707 & 0.000 & 0.707 \\
\end{array}
\]
Parsing the input file

Parsing is a process in which an input file containing “sentences” written in some language is:

- read in from a file
- tokenized
- analyzed

The semantics of the language determine the actions that are taken during the analysis. Some languages (e.g. the C programming language) are quite complex and some formal mechanisms are needed to process them. Our input language is simple enough that informal ad hoc methods suffice.

A token is a “word” in the language. In this input:

```
camera cam1
{
  pixeldim 800 600
  worlddim 8 6
  viewpoint 4 4 4
}
```

`camera, cam1, {, pixeldim, 800, 600, etc are tokens. The individual letters making up the words and the digits making up the numbers are not. If (and only if) the language is structured rigidly enough that the position in a sentence in which string values and numeric values can be known in advance, then fscanf() can be used as a combination reader/tokenizer.

- %%s token is a string of 1 or more characters
- %%d token is an integer value
- %%lf token is a double precision value

This will be the case for the raytracer.
Model description language

An example sentence in the model description language is:

```plaintext
camera cam1
{
  pixeldim 800 600
  worlddim 8 6
  viewpoint 4 4 4
}
```

- Each "sentence" in our language begins with an entity-type identifier.
- Our entity-types will include camera, material, light, spotlight, plane, sphere, etc.
- The entity-type is followed by user defined and arbitrary entity-name.
- entity-types are analogous to data types in C (int, float, double)
- entity-names are analogous to variable names in C (max, min, pixel)
- The entity-name is followed by a collection of entity-attributes enclosed in { }.
- Each entity-attribute consists of an attribute-type specifier followed by attribute-values.

- The entity-types and attribute-types are predefined keywords and will always be spelled as shown.
- The attribute-values will always follow the attribute type.
- The number of values of a particular attribute will never vary.
- The entity-attributes of any entity may appear in any order.
Attribute values map to the data structure associated with a particular entity-type in an obvious way:

camera cam1
{
  pixeldim  800 600
  worlddim  8 6
  viewpoint 4 4 6
}

The attribute values map to the `camera_t` structure in a reasonably obvious way:

typedef struct camera_type
{
  int    cookie;          /* ID's this as a camera         */
  char   name[NAME_LEN];  /* User selected camera name     */
  int    pixel_dim[2];    /* Projection screen size in pix */
  double world_dim[2];    /* Screen size in world coords   */
  vec_t  view_point;      /* Viewpt Loc in world coords    */
  irgb_t *pixmap;         /* Build image here              */
}  camera_t;

**Note:** There is no direct correspondence between an attribute name in the model description language and the variable name used to hold the attribute values.
In summary, our model description language looks informally like:

```
entity-type entity-name
{
    attribute-type attribute-value(s)
    attribute-type attribute-value(s)
    :
}

entity-type entity-name
{
    attribute-type attribute-value(s)
    :
}

:
end-of-file
```

That is,

- the attribute list of each entity definition is terminated by the } token and
- the complete model definition is terminated by end-of-file.

Use of the model description in the ray tracer.

- There is a one-to-one correspondence between sentences in the model description language and instances of structures (or structure hierarchies) in the executing raytracer.

- For each sentence read, a new structure must be dynamically created and attributes of the structure filled in using the attribute-name / attribute value pairs supplied in the model description.

- We call the process of digesting the model description and producing the structure instances parsing.
A hierarchical parser

We will use a two level hierarchy for parsing the model description language:

- The top level parser will be responsible for consuming entity-type names
- The entity-type name will identify the type of object (camera, material, light, etc) that must be constructed.
- It will then invoke a constructor for the object to be created.
- The constructor will allocation the required structure read the attribute values into it.
Constructors and parsing

In object oriented languages, each object type has an associated constructor function that is called each time a new instance of the object type is created. Therefore, if a raytracing model contains two planes, the plane constructor will be invoked twice.

*Each entity-type will provide a constructor function that will know about the attributes that apply to the entity will and be responsible for parsing them.*

Entity constructors should use the assert() mechanism to abort the program whenever:

- an unknown attribute type is encountered
- the proper number of attribute values cannot be read in
- required attributes are missing

We will constrain, to some degree, the order in which attribute-types may appear.
A general attribute parser

After writing parsers for the camera, material, and plane, the typical programmer will find repeatedly rewriting (almost) the same code tiresome and tedious and will seek a better way. It is not bad that the ad hoc approach was used initially. The very use of the ad hoc helps the programmer see common aspects of the problem and develop a more general solution. As usual we try to make our solution data driven to the extent possible.

To build a general parser we will build upon the capability of the table_lookup mechanism but replace the old table of attribute names with new tables that contain not only the attribute name but also sufficient information to allow the general parser to load the values. Specifically, for each attribute, we need the following information:

- How many values must be loaded (e.g. 2 for pixeldim, 3 for viewpoint)
- How many bytes of storage does each value occupy (4 for pixeldim, 8 for viewpoint).
- What format string should be used to read a value (%d for pixeldim, %lf for viewpoint)
- Where should the first value be stored?

Here we make use of the fact that we know adjacent array elements and members of a structure such as a vec_t are stored in adjacent memory locations. For the vec_t if the location of the x component is memory address \( a \), and then the y component is at location \( a+8 \), and the z at location \( a+16 \).

The struct pparm_type

Therefore, each entity will employ a table in which each attribute is represented by a structure of the following type.

```c
/* the parse parameter structure */

typedef struct pparm_type
{
    char *attrname;    /* Attribute name */
    int numvals;       /* Number of attribute values */
    int valsize;       /* Size of attribute in bytes */
    char *fmtstr;      /* Format string to use */
    void *loc;         /* Where to store 1st attr value */
} pparm_t;
```
Building tables of attribute descriptors

For the camera entity. We build the structure as follows:

```c
static pparm_t camera_parse[] =
{
    {"pixeldim", 2, sizeof(int), "%d", 0},
    {"worlddim", 2, sizeof(double), "%lf", 0},
    {"viewpoint", 3, sizeof(double), "%lf", 0}
};
#define NUM_ATTRS (sizeof(camera_parse) / sizeof(pparm_t))
```

Items to note:
- `camera_parse` is an array of three elements
- each element is a structure of type `pparm_t`
- initializers should be enclosed in `{ }`
- the location where the first attribute value should be stored will live in the `camera_t` structure that is eventually allocated with `malloc()`. These values can't be set until after the `camera_t` is `malloc'd`.

For the other entities such as the material entity, the structure is analogous.

```c
static pparm_t mat_parse[] =
{
    {"ambient", 3, sizeof(double), "%lf", 0},
    {"diffuse", 3, sizeof(double), "%lf", 0},
    ;
};
#define NUM_ATTRS (sizeof(mat_parse) / sizeof(pparm_t))
```
The interface to the general attribute parser

This version of `camera_init()` demonstrates how the use of the general parser can reduce your workload.

```c
void camera_init(
    FILE *in,
    model_t *model,
    int attrmax)
{
    /* malloc the camera structure */
    camera_t *cam = malloc(sizeof(camera_t));
    assert(cam != NULL);
    cam->cookie = CAM_COOKIE;

    /* Read camera name and { */
    fscanf(in, "%s", cam->name);
    fscanf(in, "%s", buf);
    assert(buf[0] == '{');

    /* Store locations where attribute data should be read */
    camera_parse[0].loc = &cam->pixel_dim;
    camera_parse[1].loc = &cam->world_dim;
    camera_parse[2].loc = &cam->view_point;

    /* Invoke the parser */
    mask = parser(in, camera_parse, NUM_ATTRS, 0);

    /* verify required attributes read */
    assert(mask == 7);

    /* remember address of camera structure */
    model->cam = cam;
}
```
/**/  
/* Generalized attribute parser */  
/* It returns a bit mask in which each possible attribute */  
/* is represented by a bit on exit the attributes that */  
/* have been found will have their bit = 1 */  

int parser(  
FILE   *in,  
pparm_t *pct,  
int      numattrs,  
int      attrmax)  
{

char attrname[NAME_LEN];  
int attrcount = 0;  
int mask = 0;  
int ndx;  

/* One trip is made through this loop for every attribute */  
/* processed... Exit from the loop is triggered by '}' */  
/* or if the maximum number of attributes is set, when */  
/* the maximum number have been processed */

fscanf(in, "%s", attrname);  
while (strlen(attrname) && attrname[0] != '}
{

/* Process one attribute */

ndx = parser_load_attr(in, pct, numattrs, attrname);  
mask |= 1 << ndx;  
attrcount++;  

/* See if its quitting time -- */

if ((attrmax) && (attrcount == attrmax))  
break;  
*attrname = 0;  
fscanf(in, "%s", attrname);
}

if (attrmax != attrcount)  
assert(attrname[0] == '}');  
return(mask);
}
static int parser_load_attr(
    FILE    *in,
    pparm_t *pct,        /* parser control table */
    int     numattrs,    /* number of legal attributes */
    char    *attrname)   /* attribute name */
{

    pparm_t *pce;     /* Entry corresp to this attribute */
    int     count = 0;
    unsigned char *loc;  /* where to store value.. have to */
    double  *work;       /* use unsigned char for pointer */
    int     ndx;         /* arithmetic to work correctly */
    int     i;

    /* table_lookuppp is an updated version of table_lookup that */
    /* takes parse control table pointer as input. */
    ndx = table_lookuppp(pct, numattrs, attrname);
    assert(ndx >= 0);

    /* Point to the proper entry in the table */
    pce = pct + ndx;    // or pce = &pct[ndx];

    /* pce->loc points to where the first value must go */
    loc = (unsigned char *) pce->loc;
/** Attributes may have different numbers of attribute values */
/** for example the viewpoint has three but the pixeldim only */
/** has 2 values. Each iteration consumes one value */

for (i = 0; i < pce->numvals; i++)
{
    count += fscanf(in, pce->fmtstr, loc);
    // work = (double *)loc;
    // fprintf(stderr, "%s %lf \n", pce->attrname, *work);
    loc += pce->valsize;  // point to next spot
}
assert(count == pce->numvals);
return(ndx);

Exercise: Design a generic table_lookup function